DERIVATION OF THE OPTIMAL CONTROL FOR AN ALL-WEATHER AIRPLANE LANDING SYSTEM

by

Carl Hugo Dubac

United States
Naval Postgraduate School

THESIS

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Carl Hugo Dubac
Major, United States Marine Corps
B.S., University of Michigan, 1956

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ABSTRACT

Optimal control theory is used to derive a controller for the final phases of an all-weather landing in the McDonald Douglas F-4J airplane. The landing is formulated as a linear tracking problem by developing a mathematical model for the airplane which is linearized about an equilibrium flight condition, and by defining a desired state trajectory. Examples are presented which illustrate the performance of the system.

Thesis by: Carl Hugo Dubac entitled Derivation of the Optimal Control for an All-Weather Airplane Landing System.

ERRATA

Page	Line	Change
21	-	On Fig. 4 change q to $Q = q$
24	8	Insert after Eq.(3.3): where $X_\alpha \triangleq X_\omega v_o$, $X_{\dot\alpha} \triangleq X_{\dot\omega} v_o$, $Z_\alpha \triangleq Z_\omega v_o$, $Z_{\dot\alpha} \triangleq Z_{\dot\omega} v_o$, $M_\alpha \triangleq M_\omega v_o$, and $M_{\dot\alpha} = M_{\dot\omega} v_o$.
26	8	Change X_q, $X_{\dot\alpha}$, Z_q, ... to X_q, $X_{\dot\alpha}$, X_{δ_e}, Z_q, ...
35	13	Eq. (4.11) should read $\underline{\dot x}(t) = A\underline{x}(t) + B\underline{u}(t) + \underline{c}$.

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The author is indebted to Professor D. E. Kirk for originally suggesting the problem and for providing guidance, assistance, and encouragement throughout the investigation. Appreciation is also extended to Professor E. R. Rang for his valuable assistance in aerodynamic analysis and to LCDR J. R. Wilson, Jr., who provided the majority of the F-4J airplane data.

I. INTRODUCTION

A most critical portion of flight, even in an ideal operating environment, is the landing of an airplane at a desired landing site. Inclement weather at the landing site, with the corresponding reduction in visibility, further complicates this portion of the flight operation to the point that landing may become an impossibility due to visibility restrictions. The increased demand, both commercially and militarily, for flight operations under all-weather conditions has prompted a great deal of research to develop an "Automatic All-Weather Landing System" capable of automatically controlling an airplane during the landing portion of a flight. The problems encountered in such investigations, although not considered insurmountable, are vast and complicated mainly because the flight dynamics of the airplane and the surrounding environment are themselves complicated. This investigation makes some simplifying assumptions to reduce this complexity and derives the necessary controls for such an all-weather landing system, using optimal control techniques.

The problem and the simplifying assumptions are described in Chapter II. In Chapter III a general model for an airplane in the landing mode is developed, and the specific model used in this investigation is presented. Chapter IV presents the optimal control theory employed and describes the method used to modify the original state variables to make the theory applicable. The problem specifications are introduced in Chapter V, the desired trajectory is formulated, and the significance of the performance measure in obtaining realistic results is discussed. In Chapter VI the procedures used in the investigation are explained, and the cases investigated are presented. The results are discussed in Chapter VII, and the conclusions and recommendations are presented in Chapter VIII.

II. PROBLEM FORMULATION

The basic descriptions and definitions of the landing portion of flight are presented in this chapter, along with the assumptions made to reduce the complexity of the problem.

A. STANDARD WEATHER CRITERIA

The International Civil Aviation Organization (ICAO) has adopted the following set of minimum weather conditions for the automatic landing of an airplane, (Ref. 1). All distance specifications are given in meters, followed in parenthesis by approximate values in feet.

Category I. Operation down to minima of 60 meters (200 ft.) decision height (altitude) and Runway Visibility Reading (RVR) of 800 meters (2600 ft.).

Category II. Operation down to minima of 30 meters (100 ft.) decision height and a RVR of 400 meters (1200 ft.).

Category III-A. Operation to and along the surface of the runway, with external visual reference during the final phase of the landing to a RVR minimum of 200 meters (700 ft.).

Category III-B. Operation to and along the surface of the runway and taxiways, with visibility sufficient only for visual taxiing comparable to an RVR of about 50 meters (150 ft.).

Category III-C. Operation to and along the surface of the runway and taxiway without external visual reference.

The Federal Aviation Agency (FAA) has established the criteria for certification of any automatic all-weather landing system based on compliance with these categories of automatic landing weather minima. Presently no automatic systems that comply with any Category III minima have been certified. However, experimental tests have been successfully conducted (Category III-B) with a system in a C-141 airplane (Ref. 2). Several automatic control systems have been certified for Category II operation, provided that the landing site has the prerequisite ground equipment. The vast majority of automatic landings are restricted to Category I conditions.

B. GROUND EQUIPMENT AT AN ALL-WEATHER LANDING SITE

At present, only one special piece of ground equipment, an Instrument Landing Systems (ILS), is required for landing at an all-weather landing facility. The ILS provides a radio beam to establish a nominal glide path to the landing site and several locator beacons, placed along the landing track to provide distance checks. One locator beacon is positioned to provide an indication of the nominal decision altitude point for the facility. Both azimuth and elevation information are provided by the ILS. Category I and Category II automatic landing operations are presently using this ground equipment with the latter, of necessity, requiring more sophisticated ILS equipment to provide the increased accuracy needed in the radio beam. Figure 1 shows present requirements for the beam accuracy of ILS Category-II ground equipment. Several major U.S. airfields have this equipment installed and operational. Because of beam distortions due to the electromagnetic disturbances, antenna installation environments, etc., the ILS equipment is not accurate below an altitude of about 60 feet. Therefore, any automatic control of the airplane beyond Category II minima will require either additional ground equipment or an automatic system in the airplane itself to effect the final phases of the landing.

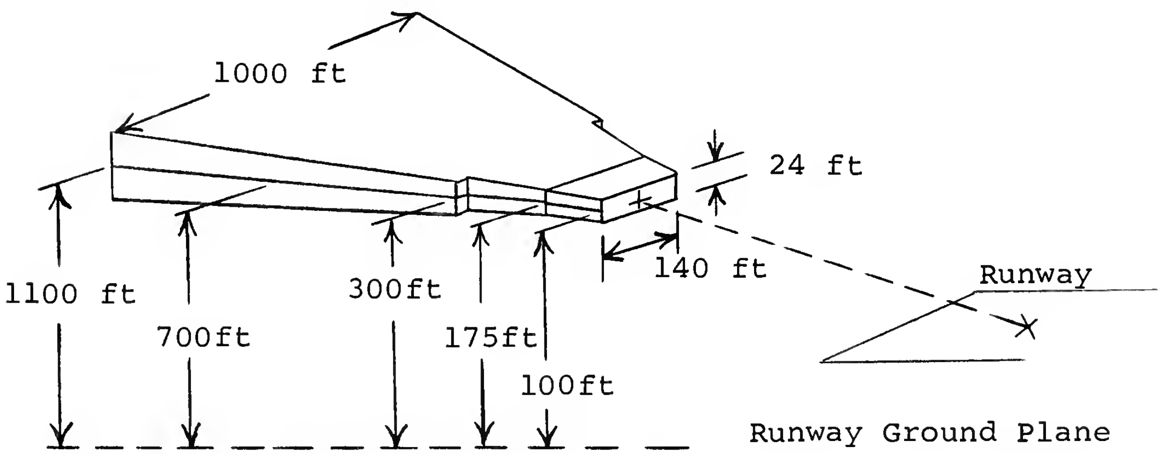

Figure 1
Category II ILS Beam Accuracy Requirements

C. PHASES OF ALL-WEATHER LANDING

The automatic landing of an airplane under all-weather conditions can be divided into five phases:

1. <u>The Approach-Intercept Phase</u>

The Approach-Intercept Phase is defined as that segment of the landing during which the configuration of the airplane is physically changed in anticipation of the landing and the airplane is directed toward the ILS beam which serves the landing site.

2. <u>The Capture-Track Phase</u>

The Capture-Track Phase is defined as that segment of the landing during which the airplane is directed toward the landing point by an automatic flight control system which uses the ILS beam as its reference.

3. <u>The Flare Phase</u>

The Flare Phase is defined as that segment of the landing during which the airplane attitude is changed in preparation for the touchdown.

4. <u>The Land Phase</u>

The Land Phase is defined as the actual touchdown of the airplane on the landing surface.

5. <u>The Roll-Out Phase</u>

The Roll-Out Phase is defined as the climax of the landing during which the airplane is slowed to a stop or, equivalently, to a safe taxi speed.

Figure 2 depicts the phases of landing in both azimuth and pitch. Included, for clarity, is the decision altitude (height), which is defined as the altitude at which a decision must be made as to whether it is feasible to continue the landing. As an example, the decision altitude for Category II landings is 100 feet above ground level.

Automatic control of the airplane during the first three phases of the landing is presently standard operating procedure for both commercial and military flight operations. However, lack of certified automatic flight control systems to accomplish the remaining phases of the landing dictates that the decision to continue the landing beyond the decision altitude be based on the premise that the pilot has visual contact with the landing site at the decision altitude, so that the airplane can be controlled manually, if necessary.

D. ASSUMPTIONS MADE FOR THE LANDING PROBLEM IN THIS STUDY

As stated in the opening remarks of this section, some simplifying assumptions are made in this study to reduce the complexity of the automatic landing problem. These assumptions are listed below.

Assumption 1. Only that portion of the landing from the decision altitude to the land phase will be considered. It is assumed that the airplane is automatically or manually

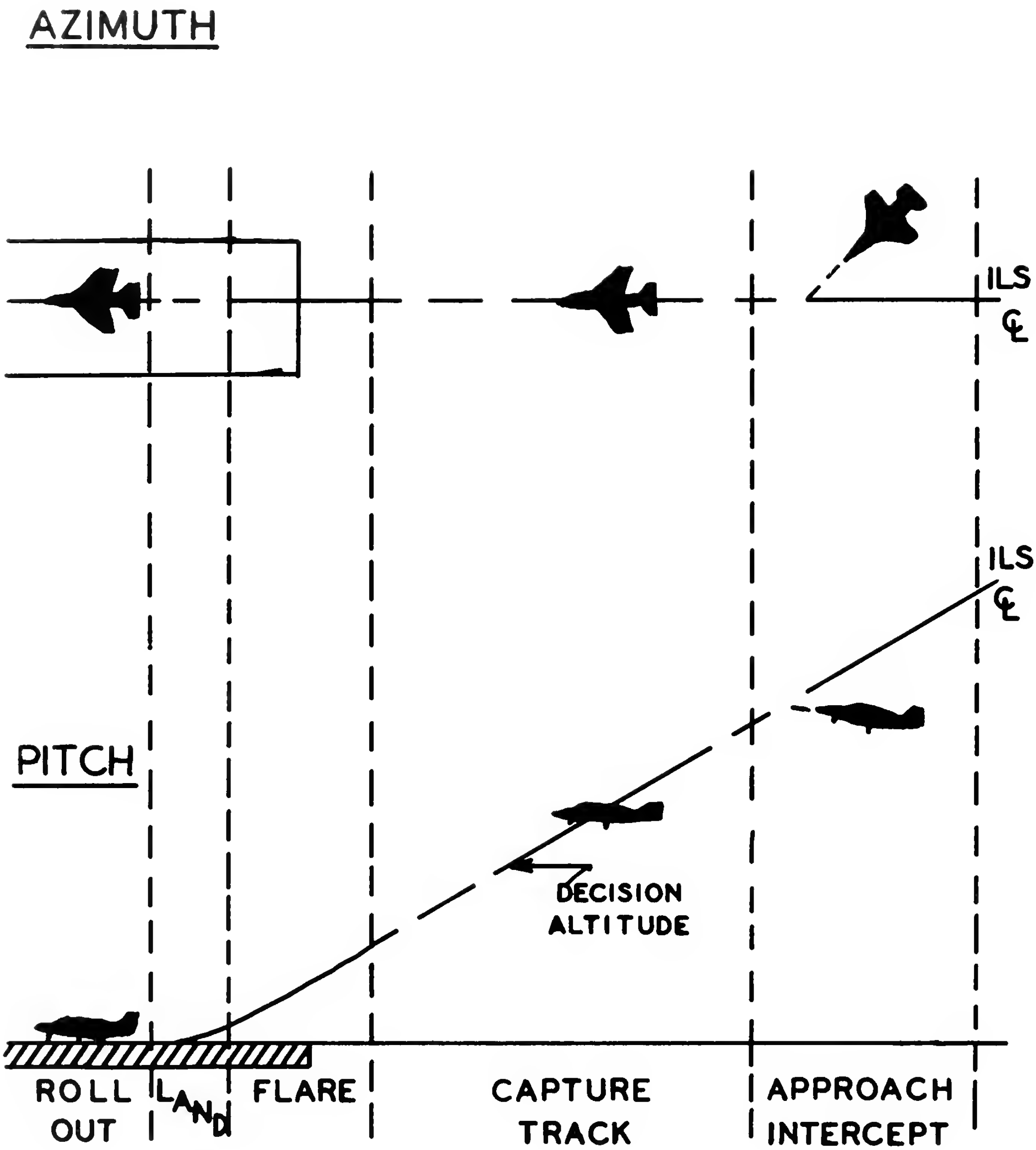

FIGURE 2
PHASES OF
AUTOMATIC LANDING

controlled, using the ILS as a reference, to the locator beacon which defines the nominal decision altitude point -- taken to be 100 feet above ground level. The roll-out phase is also assumed to be automatically or manually controlled.

Assumption 2. The airplane is physically located in space within the prescribed Category-II ILS "window" at the nominal decision altitude point and is in Equilibrium Flight (see definition in Chapter III) at that time. If these conditions are not met, it is assumed that the landing will be discontinued (airplane will be waved off).

Assumption 3. Wind effects will not be considered. During landing, the airplane is subjected to both steady-state and gusty winds, the latter being of primary importance, since the gusts are random in nature. Steady-state winds could be considered since their effect can be eliminated by a steady-state change in the airplane heading, whereas the random wind gusts could require a statistical handling of the problem. By neglecting both effects, the airplane velocity becomes equal to the ground velocity and the problem can be formulated in terms of time-to-go-to landing.

Assumption 4. Only airplane motion in the vertical plane will be considered. Lateral motion of the airplane in the final phases of landing is primarily necessary to point the airplane in the direction of the runway just prior to the actual touchdown. The airplane is normally "off" heading during the landing to counter steady-state cross-winds. Since the wind effects have been neglected, the lateral motion can be neglected.

III. MODELING THE PLANT

The first step in any control problem is the formulation of a realistic mathematical model to represent the dynamics of the plant to be controlled -- in this case, the airplane. In this chapter the formualtion of a general model applicable to any airplane is discussed, and the specific model used in this investigation is presented.

A. THE GENERAL MATHEMATICAL MODEL

Aerodynamists have developed two sets of linear equations which describe the dynamics of an airplane. These are referred to as the longitudinal or symmetric and the lateral or asymmetric equations of motion. Since, as previously stated, this problem considers only the longitudinal motions of the airplane, further reference is limited to the longitudinal equations of motion only. Derivation of these equations is beyond the scope of this study, but complete and detailed derivations can be found in the literature. (Ref. 3, 4, and 5)

Before presenting the equations of motion, it is significant, for clarification and reference purposes, to present several aerodynamic definitions and assumptions used in obtaining these equations.

1. Aerodynamic Definitions

a. Equilibrium flight is defined as unaccelerated flight. Flight is along a straight path during which the linear velocity vector measured relative to fixed space is invariant and angular velocity is zero.

b. Steady flight is defined as flight during which the linear velocity vector is invariant and angular velocity is constant. In this context, equilibrium flight is also steady flight.

c. Airplane coordinates, axes, and angles are defined in Figure 3, which shows the airplane in equilibrium flight.

V_o = Equilibrium flight linear velocity along x axis

α_o = Equilibrium flight angle of attack

θ_o = Equilibrium flight pitch angle

γ_o = Equilibrium flight glide angle

d. Instantaneous components are defined as the summation of equilibrium flight components and corresponding perturbations caused by a disturbed flight condition. Figure 4 is a representation of the airplane in a disturbed flight condition and includes the perturbations of the two control components.

V = Instantaneous linear velocity along x axis

W = Instantaneous linear velocity along z axis

Q = Instantaneous angular velocity along y axis

A = Instantaneous angle of attack

Θ = Instantaneous pitch angle

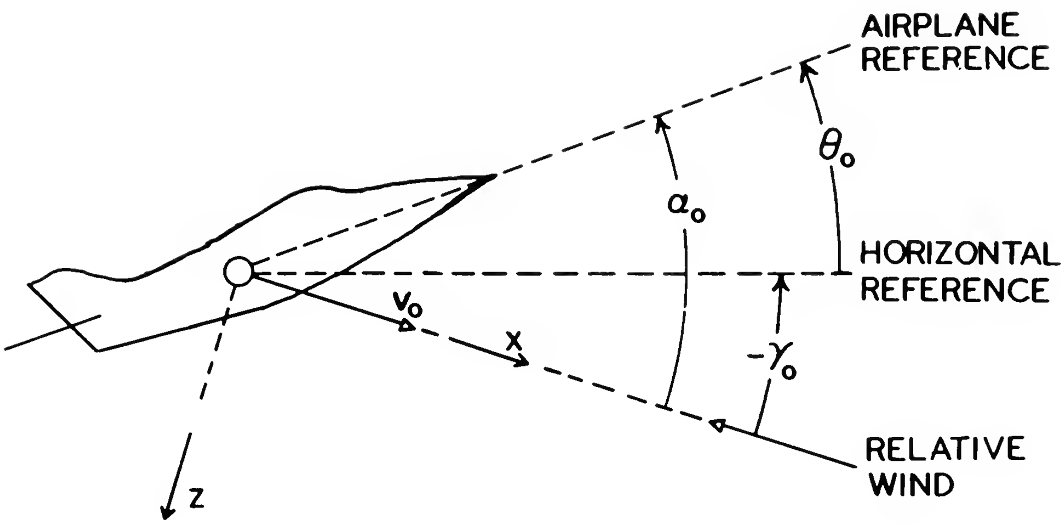

FIGURE 3

AIRPLANE IN EQUILIBRIUM FLIGHT

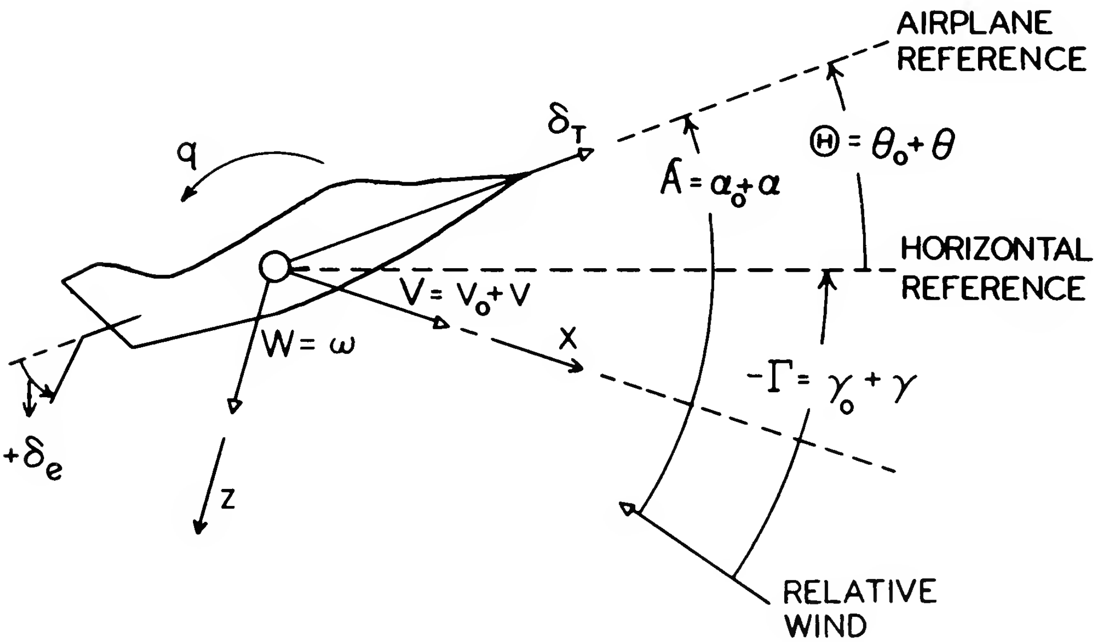

FIGURE 4

AIRPLANE IN DISTURBED FLIGHT

Γ = Instantaneous glide angle

δ_e = Elevator deflection perturbation

δ_T = Thrust perturbation

2. <u>Assumptions Used in Obtaining Equations of Motion</u>

a. The airframe is a rigid body (no aeroelastic deflection of the airframe).

b. The earth is fixed in space, and the earth's atmosphere is fixed with respect to the earth.

c. The mass of the airplane remains constant during any particular dynamic analysis.

d. The x-z plane (vertical plane) is a plane of symmetry.

e. Disturbances from steady flight are small enough so that the products and squares of changes in velocities are negligible in comparison to the changes themselves, and the disturbance angles are small enough so that the sines are equal to the angles in radians and the cosines are equal to one. Products of these angles are also approximately zero. Because these disturbances are small, the change in air density encountered by the airplane during any disturbance is considered to be zero.

f. During steady flight conditions, the airplane is assumed to be flying with wings level and all components of velocity zero except for the linear velocity along the x axis. (Equilibrium flight)

g. The flow is quasi-steady. (The air flow pattern around the airframe instantaneously changes in a steady flow pattern as the airplane changes its orientation with respect to its flight path.)

3. <u>The Longitudinal Equations of Motion</u>

$$\dot{v}(t) = X_v v(t) + X_q q(t) - g\theta(t) + X_\omega \omega(t) + X_{\dot{\omega}} \dot{\omega}(t) + X_{\delta_e} \delta_e(t) + X_{\delta_T} \delta_T(t)$$

$$\dot{\omega}(t) = Z_v v(t) + v_o q(t) + Z_q q(t) + Z_\omega \omega(t) + Z_{\dot{\omega}} \omega(t) + Z_{\delta_e} \delta_e(t) + Z_{\delta_T} \delta_T(t) \tag{3.1}$$

$$\dot{q}(t) = M_v v(t) + M_q q(t) + M_\omega \omega(t) + M_{\dot{\omega}} \dot{\omega}(t) + M_{\delta_e} \delta_e(t) + M_{\delta_T} \delta_T(t)$$

where the subscripted X, Z, and M are the Dimensional Stability Derivatives of the airplane which are parameters for a specific airplane in a specific flight regime (e.g., landing regime, subsonic regime, etc.).

By substituting the relationships

$$\begin{aligned} \omega(t) &\cong v_o \alpha(t) \\ q(t) &= \dot{\theta}(t) \end{aligned} \tag{3.2}$$

the equations of motion can be rewritten in the more familiar terms of v, α, and θ which follow:

$$\dot{v}(t) = X_v v(t) - g\theta(t) + X_q\dot{\theta}(t) + X_\alpha \alpha(t) + X_{\dot{\alpha}}\dot{\alpha}(t) + X_{\delta_e}\delta_e(t) + X_{\delta_T}\delta_T(t)$$

$$\begin{aligned}\dot{\alpha}(t) &= \frac{Z_v}{V_o} v(t) + \dot{\theta}(t) + \frac{Z_q}{V_o}\dot{\theta}(t) + \frac{Z_\alpha}{V_o}\alpha(t) \\ &+ \frac{Z_{\dot{\alpha}}}{V_o}\dot{\alpha}(t) + \frac{Z_{\delta_e}}{V_o}\delta_e + \frac{Z_{\delta_T}}{V_o}\delta_T\end{aligned} \tag{3.3}$$

$$\ddot{\theta}(t) = M_v v(t) + M_q\dot{\theta}(t) + M_\alpha\alpha(t) + M_{\dot{\alpha}}\dot{\alpha}(t) + M_{\delta_e}\delta_e(t) + M_{\delta_T}\delta_T(t)$$

In a rigorous mathematical sense, these longitudinal equations of motion, which deal with perturbations from equilibrium flight, are applicable only to infinitesimal disturbances; however, aerodynamic experience has shown that quite accurate results can be obtained by applying these equations to disturbances of finite, non-zero magnitude.

Since this problem deals with the landing approach, two additional parameters must be considered. One, the altitude of the airplane, is of primary importance, and an equation relating actual airplane altitude to the flight dynamics is required. Such a relationship can be approximated by

$$\dot{h}(t) \cong v_o \Gamma(t) \tag{3.4}$$

where h is defined as the instantaneous vertical distance above the ground of the aircraft wheels. By definition (see Figure 4)

$$\begin{aligned} \Gamma(t) &= \gamma_o + \gamma(t) \\ \gamma(t) &= \theta(t) - \alpha(t) \end{aligned} \tag{3.5}$$

and therefore,

$$\dot{h}(t) = v_o \gamma_o + v_o(\theta(t) - \alpha(t)) \tag{3.6}$$

The second parameter is ground effect -- defined as the effect on the airplane dynamics as the airplane nears the vicinity of the stationary plane (the ground). This effect on an airplane is non-linear and is dependent on the size and shape of the specific airplane.

The formulation of a general mathematical model to describe the motions of any airplane in the landing approach is now complete. All that remains is application to a specific airplane.

B. THE SPECIFIC MATHEMATICAL MODEL

The main problem associated with the use of the general mathematical model is determination of the numerical values for the Dimensional Stability Derivatives for the specific airplane. Because of the vast amount of experimental data available on the airplane, both from wind tunnel studies and actual flight testing, the McDonald Douglas F-4J Phantom II jet fighter was selected. Appendix A contains representative data for this airplane in the landing configuration.

As the F-4J Dimensional Stability Derivatives, X_q, $X_{\dot{\alpha}}$, Z_q, $Z_{\dot{\alpha}}$, and M_{δ_T} are zero and ground effects are neglected, the longitudinal equations of motion and the height equation can be simplified as follows:

$$\dot{v}(t) = X_v v(t) + X_\alpha \alpha(t) - g\theta(t) + X_{\delta_T}\delta_T(t)$$

$$\dot{\alpha}(t) = \frac{Z_v}{v_o} v(t) + \frac{Z_\alpha}{v_o}\alpha(t) + \dot{\theta}(t) + \frac{Z_{\delta_e}}{v_o}\delta_e(t) + \frac{Z_{\delta_T}}{v_o}\delta_T(t)$$

$$\ddot{\theta}(t) = M_v v(t) + M_\alpha \alpha(t) + M_{\dot{\alpha}}\dot{\alpha}(t) + M_q \dot{\theta}(t) + M_{\delta_e}\delta_e(t) \tag{3.7}$$

$$\dot{h}(t) = v_o(\theta(t) - \alpha(t)) + v_o\gamma_o$$

Defining

$$\underline{x} \triangleq \begin{pmatrix} v \\ \alpha \\ \theta \\ \dot{\theta} \\ h \end{pmatrix} \tag{3.8}$$

and

$$\underline{u} \triangleq \begin{pmatrix} \delta_e \\ \delta_T \end{pmatrix} \tag{3.9}$$

the equations are manipulated into the state variable form

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t) + C \tag{3.10}$$

with the result

$$\dot{\underline{x}}(t) = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{21} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ a_{41} & a_{42} & 0 & a_{44} & 0 \\ 0 & a_{52} & a_{53} & 0 & 0 \end{pmatrix} \underline{x}(t)$$

$$+ \begin{pmatrix} 0 & b_{12} \\ b_{21} & b_{22} \\ 0 & 0 \\ b_{41} & b_{42} \\ 0 & 0 \end{pmatrix} \underline{u}(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ c_{51} \end{pmatrix} \tag{3.11}$$

where

$$a_{11} = X_v \qquad a_{52} = -v_o$$

$$a_{12} = X_\alpha \qquad a_{53} = v_o$$

$$a_{13} = -g \qquad b_{12} = X_{\delta_T}$$

$$a_{21} = \frac{Z_v}{v_o} \qquad b_{21} = \frac{Z_{\delta_e}}{v_o}$$

$$a_{22} = \frac{Z_\alpha}{v_o} \qquad b_{22} = \frac{Z_{\delta_T}}{v_o}$$

$$a_{41} = M_v + \frac{M_{\dot{\alpha}} Z_v}{v_o} \qquad b_{41} = M_{\delta_e} + \frac{M_{\dot{\alpha}} Z_{\delta_e}}{v_o}$$

$$a_{42} = M_\alpha + \frac{M_{\dot{\alpha}} Z_\alpha}{v_o} \qquad b_{42} = \frac{M_{\dot{\alpha}} Z_{\delta_T}}{v_o}$$

$$a_{43} = M_q + M_{\dot{\alpha}} \qquad c_{51} = v_o \gamma_o$$

Appendix B contains the numerical values for these elements.

Heretofore in the formulation, it was assumed that the two perturbation controls, elevator deflection and thrust, are instantaneously available to control the airplane. In reality, both controls have associated dynamics, which are aerodynamically referred to as elevator actuator lag and lag between the thrust command and thrust. These effects can be approximated by use of the following differential equations:

$$\begin{aligned} \dot{\delta}_e(t) &= \frac{1}{\tau_{\delta_e}} \left(\delta_{e_c}(t) - \delta_e(t)\right) \\ \dot{\delta}_T(t) &= \frac{1}{\tau_{\delta_T}} \left(\delta_{T_c}(t) - \delta_T(t)\right) \end{aligned} \tag{3.12}$$

where the subscripted τ's are the appropriate component time constants and δ_{e_c} and δ_{T_c} are the elevator deflection and thrust commands respectively. In this investigation it will be assumed that the controls are instantaneously available; i.e., the dynamics will be neglected.

This completes the mathematical modeling of the F-4J airplane in the landing configuration. The model was tested on an IBM System/360 digital computer, using a Fourth-Order Runge-Kutta routine to solve the differential equations. The responses resulting from various control inputs were consistent with actual airplane responses.

IV. OPTIMAL CONTROL THEORY

This chapter will not attempt to present the entire field of optimal control theory, but rather will assume that the reader is familiar with it; hence, only those techniques pertinent to this investigation will be discussed. Some definitions will first be presented, followed by mathematical techniques employed to obtain the optimal control.

A. OPTIMAL CONTROL DEFINITIONS

1. Control History

The history of control input values during the time interval $[t_o, t_f]$ is denoted by $\underline{u}$.

2. State Trajectory

The history of state values during the time interval $[t_o, t_f]$ is denoted by $\underline{x}$.

3. Admissible Control

U denotes the set of all control histories which satisfy the physical control constraints during the time interval $[t_o, t_f]$.

4. Admissible Trajectory

X denotes the set of all state trajectories which satisfy the physical state variable constraints during the time interval $[t_o, t_f]$.

5. <u>The Performance Measure</u>

J denotes a scalar measure of the performance of a system when a control history is applied.

6. <u>Optimal Control</u>

A control which causes the system to follow an admissible trajectory which minimizes the performance measure is called an optimal control and is denoted by $\underline{u}^*$.

7. <u>Optimal Trajectory</u>

The admissible trajectory which results when an optimal control is applied to the system is called an optimal trajectory and is denoted by $\underline{x}^*$.

8. <u>Tracking Problem</u>

A problem wherein the intent is to maintain the state trajectory as close as possible to a desired trajectory -- denoted by $\underline{r}$ -- in the interval $[t_o, t_f]$ is called a tracking problem.

B. THE LINEAR TRACKING PROBLEM

Since the automatic landing problem as formulated in Chapter II is directed toward controlling the airplane in a desired manner during the final phases of a landing, and the mathematical model of the airplane as presented in Chapter III is linear, the problem can be considered as a linear tracking problem.

It has been shown (Ref. 6 and 7), where complete derivations can be found, that, given a set of state equations

$$\dot{\underline{x}}(t) = A(t)\ \underline{x}(t) + B(t)\underline{u}(t) \tag{4.1}$$

where

$\underline{x}(t)$ is the state vector of dimension n

$\underline{u}(t)$ is the control vector of dimension m

$A(t)$ is a nxn matrix

$B(t)$ is a nxm matrix

and the performance measure

$$J = \frac{1}{2}\ \|\underline{x}(t_f) - \underline{r}(t_f)\|_H^2 + \frac{1}{2}\int_{t_o}^{t_f}\left[\|\underline{x}(t) - \underline{r}(t)\|_{Q(t)}^2 + \|\underline{u}(t)\|_{R(t)}^2\right]dt \tag{4.2}$$

where

$$\|\underline{x}(\tau) - \underline{r}(\tau)\|_{Q(\tau)}^2 \triangleq \left[\underline{x}(\tau) - r(\tau)\right]^T Q(\tau)\left[\underline{x}(\tau) - \underline{r}(\tau)\right],$$

the final time t_f is fixed, $\underline{x}(t_f)$ is free, $\underline{u}(t)$ is unconstrained, H and $Q(t)$ are real symmetric positive semi-definite matrices, and $R(t)$ is a real symmetric positive definite matrix, that the optimal control exists and is unique. The optimal control is given by

$$\underline{u}^*(t) = F(t)\ \underline{x}^*(t) + \underline{g}(t) \tag{4.3}$$

where F(t) is the mxn matrix of feedback gains, and g(t) is the mxl command signal vector which is dependent on the system parameters and the desired state trajectory. Figure 5 is a block diagram of the plant and optimal controller.

F(t) and g(t) are given by

$$F(t) = - R(t)^{-1} B^T(t)K(t) \tag{4.4}$$

$$\underline{g}(t) = - R(t)^{-1} B^T(t)\ \underline{s}(t) \tag{4.5}$$

where K(t) is the solution of the Riccati-type matrix differential equation

$$\begin{aligned}\dot{K}(t) = &- K(t)A(t) - A^T(t)K(t) - Q(t) \\ &+ K(t)B(t)R^{-1}(t)B^T(t)K(t)\end{aligned} \tag{4.6}$$

with boundary conditions

$$K(t_f) = H \tag{4.7}$$

and $\underline{s}(t)$ is the solution of the linear vector differential equation

$$\dot{\underline{s}}(t) = - [A^T(t)-K(t)B(t)R^{-1}(t)B^T(t)]\underline{s}(t)+Q(t)\underline{r}(t) \tag{4.8}$$

with boundary conditions

$$\underline{s}(t_f) = - H\underline{r}(t_f) \tag{4.9}$$

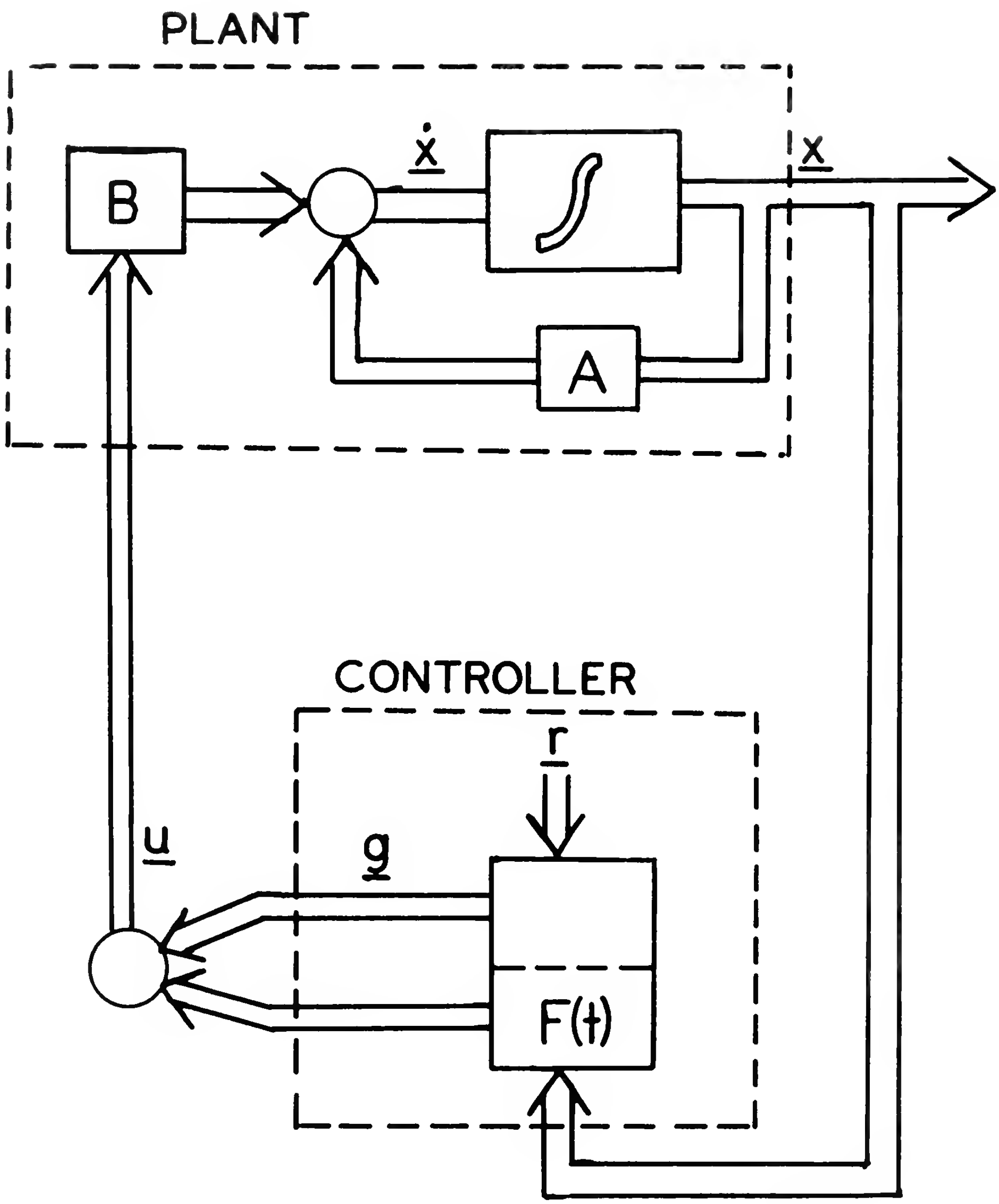

FIGURE 5
BLOCK DIAGRAM OF PLANT AND
CONTROLLER
IN A LINEAR TRACKING PROBLEM

Since the boundary conditions at the final time t_f are known, these two matrix differential equations must be integrated backwards in time from t_f to t_o.

It was assumed that all states of the system can be measured.

C. MODIFICATIONS TO SPECIFY STATE PLANT

The previous discussion indicated that the optimal control in a linear tracking problem exists and can be obtained, assuming that the plant is of the form

$$\dot{\underline{x}}(t) = A(t)\underline{x}(t) + B(t)\underline{u}(t) \tag{4.10}$$

Since the plant model, as presented in Chapter III, is in the form

$$\dot{\underline{x}}(t) + A\underline{x}(t) + B\underline{u}(t) + \underline{c} \tag{4.11}$$

some modifications are necessary in order to adapt the specific problem to the theory presented.

This modification can be accomplished by observing that the $\underline{c}$ vector contains only one non-zero element, i.e., the differential equation describing the altitude of the airplane

$$\dot{h}(t) = v_o\gamma_o + v_o(\theta(t) - \alpha(t)) \tag{4.12}$$

contains the constant term $v_o\gamma_o$.

By defining

$$\bar{h}(t) \triangleq h(t) - h_e(t) \tag{4.13}$$

where

$$h_e(t) \triangleq h(0) + v_o \gamma_o t \tag{4.14}$$

is the nominal equilibrium altitude of the airplane during the interval $[t_o, t_f]$. It follows that

$$\dot{h}_e(t) = v_o \gamma_o \tag{4.15}$$

$$\dot{\bar{h}}(t) = \dot{h}(t) - \dot{h}_e(t) \tag{4.16}$$

$$\dot{\bar{h}}(t) = v_o \gamma_o + v_o(\theta(t) - \alpha(t)) - v_o \gamma_o \tag{4.17}$$

$$\dot{\bar{h}}(t) = v_o(\theta(t) - \alpha(t)) \tag{4.18}$$

So $\bar{h}(t)$, as defined, is the perturbation of the altitude of the airplane about the nominal equilibrium altitude.

With this modification, a new set of states, hereafter referred to as the revised states (to distinguish them from the actual states), can be defined as

$$\underline{\bar{x}} = \begin{pmatrix} v \\ \alpha \\ \theta \\ \dot{\theta} \\ \bar{h} \end{pmatrix} = \underline{x} - \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ h_e \end{pmatrix} \qquad (4.19$$

and the revised state equations become

$$\dot{\bar{\underline{x}}}(t) = A\bar{\underline{x}}(t) + B\underline{u}(t) \tag{4.20}$$

where the elements of A and B remain identical to those previously formulated in Chapter III, equation (3.11). This revised plant is now in the proper form for application of the optimal control theory. The solution then becomes

$$\underline{u}^*(t) = F(t)\bar{\underline{x}}^*(t) + \underline{g}(t) \tag{4.21}$$

from which the behavior of the actual states can be determined by substituting the definition of $\bar{\underline{x}}(t)$. This substitution produces the optimal control

$$\underline{u}^*(t) = F(t)\underline{x}^*(t) - F(t)\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ h_e(t) \end{pmatrix} + \underline{g}(t) \tag{4.22}$$

By this simple mathematical manipulation, the problem can be solved by applying optimal control theory to the revised state equations and then expressing the results in terms of the actual states.

V. SPECIFICATIONS, DESIRED TRAJECTORIES, PERFORMANCE MEASURE

In this chapter the applicable specifications are introduced and a set of desired state trajectories for the problem are derived. Many of the results presented are based on the author's experience as a pilot operating the F-4 and other jet airplanes. The final section of this chapter discusses the significance of the performance measure in obtaining realistic results.

A. PROBLEM SPECIFICATIONS

A successful automatic landing requires that certain conditions relative to the airplane and its environment be satisfied. These conditions, which assume the role of specifications for the problem, are formulated in terms of performance requirements and constraints on the system states and controls, together with specifications regarding related parameters. The following specifications are of primary importance to the problem under consideration.

1. Velocity

The instantaneous velocity of the airplane during the landing portion of flight must remain above 1.1 V_{stall} or $V(t) \geq 214.5$ ft/sec. The upper velocity limit depends on the structural limit with landing gear and flaps extended; flight experience indicates that a reasonable upper limit is 1.1 v_o or $V(t) \leq 245$ ft/sec. In terms of the specified state, $v(t)$, which is the perturbation velocity about equilibrium flight, these limitations are

$$-8.5 \text{ ft/sec} \leq v(t) \leq 22 \text{ ft/sec} \quad , \quad t \epsilon [t_o, t_f] \ .$$

2. Angle of Attack

The angle of attack must remain below 0.9 $\mathcal{A}_{stall}$ or $\mathcal{A}(t) \leq 0.44$ radians. No stringent lower limit exists but, as in the velocity case, experience indicates that a reasonable minimum value is approximately $0.75\alpha_o$ or $\mathcal{A}(t) \geq 0.25$ radians. As a result, the perturbation angle of attack limits were established as

$$|\alpha(t)| \leq 0.11 \text{ rad} \quad , \quad t \epsilon [t_o, t_f] \ .$$

3. Pitch Angle

During the landing portion of flight, the instantaneous pitch angle is closely related to the angle of attack but does not have stringent limitations on its excursions. However, at the actual touchdown point, limitations do exist primarily to prevent the airplane from either landing nose wheel first (low pitch angle) or tail first (high pitch angle). For the F-4J airplane these limitations, in terms of the perturbation pitch angle from equilibrium flight conditions, were established as

$$-0.20 \text{ rad} \leq \theta(t) \leq 0.25 \text{ rad} \quad , \quad t \epsilon [t_o, t_f] \ .$$

4. Pitch-Angle Rate

Flight experience has indicated that this parameter should be held to a minimum for pilot comfort. (See discussion under Related Specifications, below.) Realistic limits for the pitch angle rate were established as

$$|\dot{\theta}(t)| \leq 0.08 \text{ rad/sec} \quad , \quad t\epsilon[t_o, t_f] \ .$$

3. Altitude

In the problem formulation, Chapter II, it was assumed that the altitude of the airplane at the initial time was within the prescribed Category II ILS "window" or

$$88 \text{ ft} \leq h(t_o) \leq 112 \text{ ft}$$

In terms of perturbations about equilibrium flight conditions, this limitation can be expressed as

$$|\bar{h}(t_o)| \leq 12 \text{ ft}$$

As the airplane approaches the actual touchdown point, however, altitude excursions about desired conditions should satisfy more stringent limits. As a result, the following limitations were established for altitude perturbations about a desired altitude trajectory, in the intervals indicated. (See Section B, this chapter, for desired trajectory.)

In the interval from the decision altitude to the flare initiation altitude

$$|\bar{h}(t) - \bar{h}_d(t)| \leq 12\text{ft} \ , \quad t\epsilon[t_o, t_1],$$

and in the interval from the flare initiation altitude to actual touchdown

$$|\bar{h}(t) - \bar{h}_d(t)| \le 5\text{ft}, \qquad t\epsilon[t_1, t_f] ,$$

where $\bar{h}_d(t)$ is the desired revised altitude state trajectory, and t_1 is the time at which the flare phase begins.

6. Elevator Deflection

Physical limits for elevator travel exist, and for the F-4J airplane in equilibrium flight in a landing approach, were established as

$$-0.26 \text{ rad} \le \delta_e(t) \le 0.22 \text{ rad}, \qquad t\epsilon[t_o, t_f] .$$

7. Thrust

Physical limits on thrust available exist for the F-4J airplane and are dependent on the selection of power; i.e., MILITARY engine operation or AFTER BURNER (A/B) engine operation. Flight experience has shown that realistic limits for thrust perturbations about equilibrium flight conditions in the landing portion of flight can be established as

$$|\delta_T(t)| \le 3000 \text{ lbs}, \qquad t\epsilon[t_o, t_f] .$$

This range is well within the thrust-available range of the F-4J airplane without A/B selection.

8. Related Specifications

Several related performance requirements and constraints are applicable to the problem and are presented in the following listing:

a. The actual touchdown point must be within 150 feet of the desired touchdown point on the landing surface. Since the problem was formulated in the time domain, this restriction was established by requiring that the actual touchdown occur within ± 0.65 seconds of the desired touchdown time.

b. At the actual touchdown time the airplane must have no tendency to float, or, in other words, the airplane should have a positive rate of descent. In addition, the rate of descent at touchdown must be within the structural sink rate limits for the airplane (Appendix A). Experience indicates that realistic rate of descent limits for the F-4J at the actual touchdown time are

$$-9\text{ft/sec} \le \dot{h}(t_f) \le -3\text{ft/sec}$$

c. The normal acceleration[1] of the airplane -- defined as a perturbation from the gravitational acceleration of equilibrium flight (1.0g) -- can be approximated by

$$n(t) \cong \frac{V_o}{g}(\dot{\theta}(t) - \dot{\alpha}(t))$$

---

[1] The airplane acceleration perpendicular to the horizontal reference plane.

and has physical limits based on the structural strength of the airplane. These limits are normally not attained in the landing portion of flight, due to the low velocity and control effectiveness in this regime. However, the function n(t) gives an indication of the smoothness of the landing, and its effect can actually be felt by the pilot during the landing: therefore, to maintain the normal acceleration as closely as possible to the equilibrium flight condition of 1.0g the following limitation was established

$$|n(t)| \leq 0.2g, \qquad t\epsilon[t_o, t_f] .$$

B. DESIRED TRAJECTORY

The problem has been formulated as a tracking problem; hence the desired state trajectory must be determined. This section presents a general desired trajectory appropriate for any airplane in the landing portion of flight, and indicates how these trajectories are applied to the F-4J landing problem.

1. General Desired Trajectory

Since the altitude of the airplane is of great importance during the landing, it is an obvious starting point in the formulation of a set of desired state trajectories. In Chapter II it was stated that this investigation would consider only that portion of the landing from the decision altitude (100 feet) to the touchdown point. Referring to Figure 2, this portion of the landing includes a section of the capture-track phase, the flare phase, and the land phase. In Chapter III it was shown that the problem could be formulated in the time domain. These two considerations lead to a general altitude trajectory (specified in the time domain) consisting of a constant descent from the decision altitude at $t = t_o = 0$ to a selected flare point at $t = t_1$, followed by a flare to touchdown at $t = t_f$.

In the optimal control theory presented in Chapter IV, one of the necessary requirements was that the final time (t_f) be fixed. To alleviate the obvious disadvantage that this requirement imposes, it seemed appropriate to extend the final time beyond the actual touchdown time to ensure that a landing occurs prior to t_f. By so doing, an imaginary flare plane below the actual landing plane was established. This implies that the airplane nominally will land before the final time specified. Figure 6 depicts this general concept in graphical form, where the capture-track phase is the interval $[0, t_1)$, the actual flare phase is the interval $[t_1, t_2)$, and the actual land phase is t_2. The imaginary flare phase is the interval $[t_1, t_f)$ and the imaginary land phase is at the time t_f.

The rate of altitude change (rate of descent), $\dot{h}(t)$, must be considered in conjunction with the general altitude trajectory, even though it is not a state variable. Ideally, during the capture-track phase $[0, t_1)$, the rate of descent will be a constant, while during the flare phase, it will steadily decrease so that at the actual touchdown time (t_2), the rate of descent will be within the established limits. Utilizing this consideration, the set of equations

$$h_d(t) = h_e(t) + h(0) + v_o\gamma_o t \tag{5.1}$$

$$\dot{h}_d(t) = \dot{h}_e(t) = v_o\gamma_o \tag{5.2}$$

which describe the desired altitude trajectory, $h_d(t)$, and rate of descent, $\dot{h}_d(t)$, for the interval $[0,t_1)$ were formulated.

For the interval $[t_1,t_f]$, the desired altitude and rate of descent were defined as

$$h_d(t) = h(0) + v_o\gamma_o t + \frac{L_2}{L_1}\left(e^{L_1(t-t_1)} -1\right) -L_2(t-t_1) \tag{5.3}$$

$$\dot{h}_d(t) = v_o\gamma_o + L_2\left(e^{L_1(t-t_1)} -1\right) \tag{5.4}$$

Since the revised altitude state was defined in Chapter IV to be

$$\bar{h}(t) = h(t) - h_e(t) \tag{5.5}$$

equations (5.1) and (5.2) became

$$\bar{h}_d(t) = \dot{\bar{h}}_d(t) = 0, \qquad t\epsilon[0,t), \tag{5.6}$$

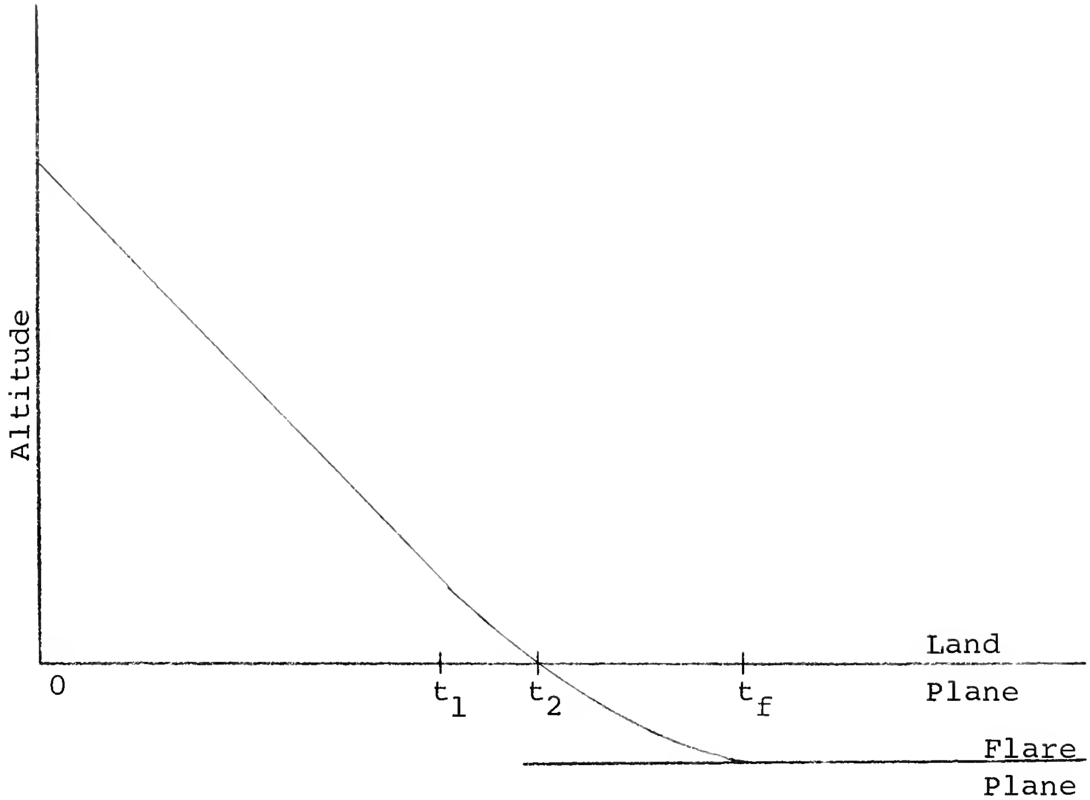

Figure 6

General Desired Altitude Trajectory

and

$$\bar{h}_d(t) = \frac{L_2}{L_1}\left(e^{L_1(t-t_1)} - 1\right) - L_2(t-t_1) \qquad (5.7)$$

$$\dot{\bar{h}}_d(t) = L_2\left(e^{L_1(t-t_1)} - 1\right), \qquad t\epsilon[t_1, t_f] \; . \qquad (5.8)$$

Equations (5.6) and (5.7) represent the desired revised altitude trajectory in the interval $[0, t_f]$ in terms of perturbations about the equilibrium altitude.

These general expressions for the desired revised rate of descent, $\dot{h}_d(t)$, provide insight into the form of the desired angle of attack trajectory, α_d, and pitch angle trajectory, θ_d, since from equation (4.18)

$$\dot{\bar{h}}(t) = v_o(\theta(t) - \alpha(t)) \quad . \tag{5.9}$$

Therefore,

$$\theta_d(t) - \alpha_d(t) = 0, \qquad t\epsilon[0,t_1) \tag{5.10}$$

and

$$\theta_d(t) - \alpha_d(t) = \frac{L_2}{v_o}\left(e^{L_1(t-t_1)} - 1\right), \quad t\epsilon[t_1,t_f]. \tag{5.11}$$

By definition (Chapter II), α and θ are zero when the airplane is in equilibrium flight, hence equation (5.10) is consistent. The problem remains to find equations for α and θ in the interval $[t_1,t_f]$ which are consistent with the requirements of equation (5.11). Before proceeding, recall that the desired pitch-angle-rate trajectory, $\dot{\theta}$ must also be considered. An acceptable expression for the desired pitch-rate trajectory is a linear function during the flare phase interval $[t_1,t_f]$ given by

$$\dot{\theta}_d(t) = L_3(t-t_1) \tag{5.12}$$

Integrating equation (5.12) and applying the desired boundary condition at $t = t_1$, i.e., $\theta_d(t_1) = 0$

$$\theta_d(t) = \frac{L_3}{2}(t - t_1)^2 . \tag{5.13}$$

Substituting equation (5.13) into (5.11) yields

$$\alpha_d(t) = \frac{L_3}{2}(t - t_1)^2 - \frac{L_2}{v_o}\left(e^{L_1(t-t_1)} - 1\right) \tag{5.14}$$

as the expression for the desired angle of attack in the interval $[t_1, t_f]$.

The final state component trajectory which must be considered is the velocity trajectory. Experience from both an operational and a safety standpoint has established that the airspeed during a landing should remain essentially constant. This implies that v_d, the desired velocity trajectory, should be zero during the entire interval of interest, hence

$$v_d(t) = 0, \quad t \epsilon [0, t_f] . \tag{5.15}$$

2. Specific Desired Trajectories

In the preceding section a general set of desired state trajectories -- applicable to any airplane -- were formulated. What remains is application to the F-4J airplane so that the unknown constant terms can be evaluated. Since, in the interval $[0, t_1)$, the desired state trajectory is zero, the following discussion will be concerned with the interval $[t_1, t_f]$ where the equations of interest are (5.3), (5.4), (5.7), (5.8), (5.12), (5.13) and (5.14) -- hereafter referred to as the reference equations.

The first consideration, however, was to determine a realistic time frame for the problem. The equilibrium airspeed, v_o, of the F-4J, as given in Appendix A, is 223 ft/sec. Assuming that the airplane is at the nominal decision altitude (100 feet) at $t = 0$ and that the airplane remained in equilibrium flight on a negative three-degree glide slope, the actual time of touchdown from equation (4.14) is approximately 8.57 seconds. This assumes that there is no flare phase. Although the standard operating procedure in the U.S. Navy is to operate the F-4J airplane with no flare, this investigation will include a flare which commences at $t_1 = 6$ seconds; at this time the airplane is at a nominal altitude of approximately 30 feet. It follows then that a reasonable time frame for the problem is 10 seconds, and that the interval $[t_1, t_f]$ is 4 seconds in duration.

Next, values for the unknown constants in the reference equations must be obtained. Boundary conditions for the reference equations at $t = t_1 = 6$ seconds are known, and final-time boundary conditions can be selected to provide realistic trajectories. However, recalling that the actual touchdown of the airplane occurs prior to the final time, and that the specifications refer to this actual touchdown time, another set of boundary conditions at $t = t_2$ must be satisfied. The conditions at $t = t_2$ are not exactly given, but realistic values can be selected to meet the necessary specifications. As a result, values for the unknown constants L_1, L_2, and L_3 of the reference equations were found by a trial-and-error method. Values of these constants were arbitrarily chosen to satisfy boundary conditions at $t = t_f$ and then substituted into the reference equations to obtain the actual touchdown time, t_2, and the corresponding values of the parameters at $t = t_2$. This process was continued until values of L_1, L_2, and L_3 were found, which resulted in realistic trajectories. The values

$$L_1 = 0.25$$

$$L_2 = 4.68$$

$$L_3 = 0.016$$

which were obtained result in a desired actual touchdown time, (t_2), of 9.3 seconds. The reference equations which apply to the interval [6,10] are then

$$h_d(t) = h(0) + v_o\gamma_o t + \frac{4.68}{0.25}\left(e^{.25(t-6)}-1\right) - 4.68(t-6) \tag{5.16}$$

$$\dot{h}_d(t) = v_o\gamma_o + 4.68\left(e^{.25(t-6)}-1\right) \tag{5.17}$$

$$\bar{h}_d(t) = \frac{4.68}{0.25}\left(e^{.25(t-6)}-1\right) -4.68(t-6) \tag{5.18}$$

$$\dot{\bar{h}}_d(t) = 4.68 \left(e^{.25(t-6)}-1\right) \tag{5.19}$$

$$\dot{\theta}_d(t) = 0.016(t-6) \tag{5.20}$$

$$\theta_d(t) = 0.008(t-6)^2 \tag{5.21}$$

$$\alpha_d(t) = 0.008(t-6)^2 - \frac{4.68}{v_o}\left(e^{.25(t-6)}-1\right), \tag{5.22}$$

while in the interval $[0,6)$ the applicable equations are

$$h_d(t) = h(0) + v_o\gamma_o t \tag{5.23}$$

$$\dot{h}_d(t) = v_o\gamma_o \tag{5.24}$$

$$\bar{h}_d(t) = \dot{\bar{h}}_d(t) = \dot{\theta}_d(t) = \theta_d(t) = \alpha_d(t) = 0 \tag{5.25}$$

and

$$v_d(t) = 0, \quad t\epsilon[0,10] . \tag{5.26}$$

Table A summarizes the solutions of equations (5.16) through (5.22) at the flare initiation time (t_1), the actual touch-down time (t_2), and the final time (t_f).

Table A

Solutions to Reference Equations at Flare Initiation Time, Actual Touchdown Time, and Final Time

Parameter	Equation No.		Solutions at	
		t=6 sec	t=9.3 sec	t=10 sec
h_d(ft)	5.16	29.947	0	-3.286
$\dot{h}_d$(ft/sec)	5.17	-11.676	-5.667	-3.621
$\bar{h}_d$(ft)	5.18	0	8.582	13.469
$\dot{\bar{h}}_d$(ft/sec)	5.19	0	6.009	8.055
$\dot{\theta}_d$(rad/sec)	5.20	0	0.053	0.064
θ_d(rad)	5.21	0	0.087	0.128
α_d(rad)	5.22	0	0.060	0.092

Figure 7 presents a plot of the actual desired altitude trajectory with the equilibrium altitude trajectory included for comparison. Figure 8 illustrates the revised desired altitude trajectory h_d, while Figure 9 depicts the three angular components of the desired state trajectory, α_d, θ_d, and $\dot{\theta}_d$ and the velocity trajectory v_d.

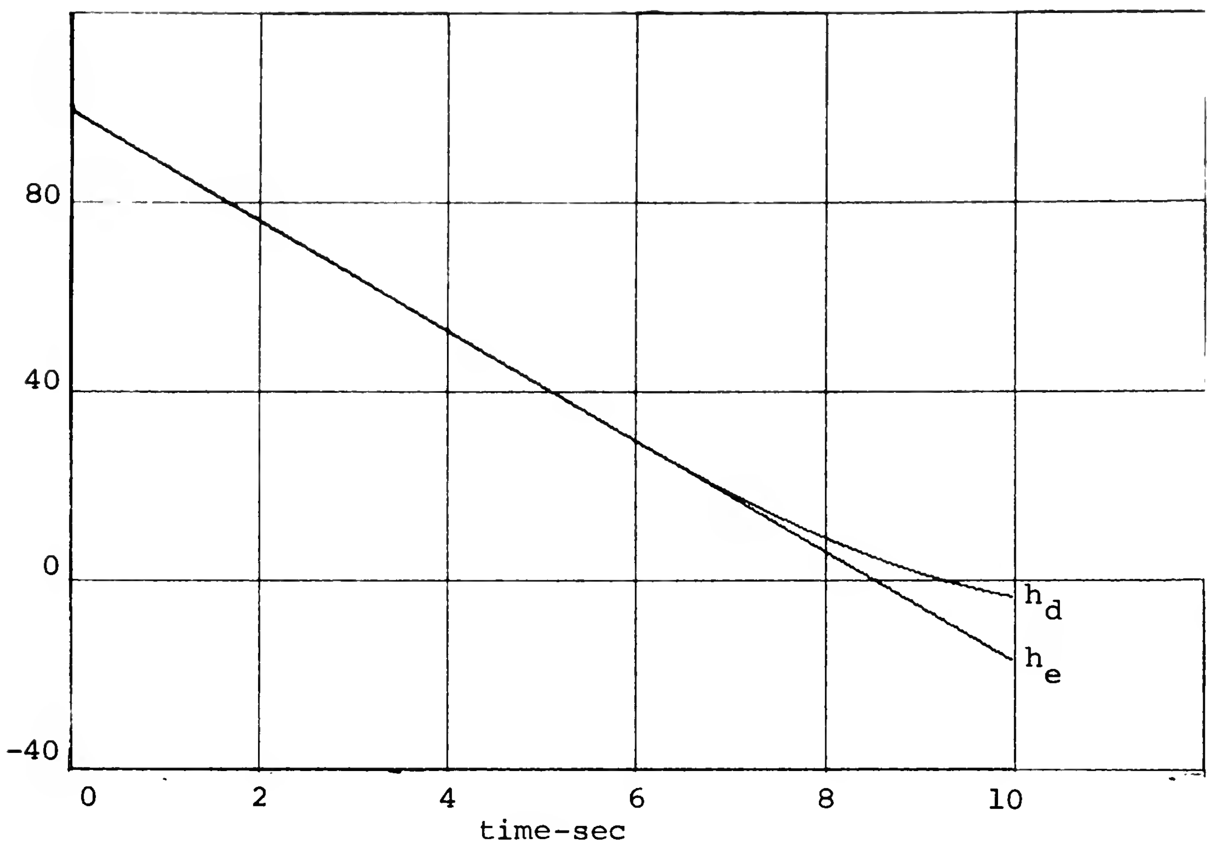

Figure 7

Desired Altitude Trajectory

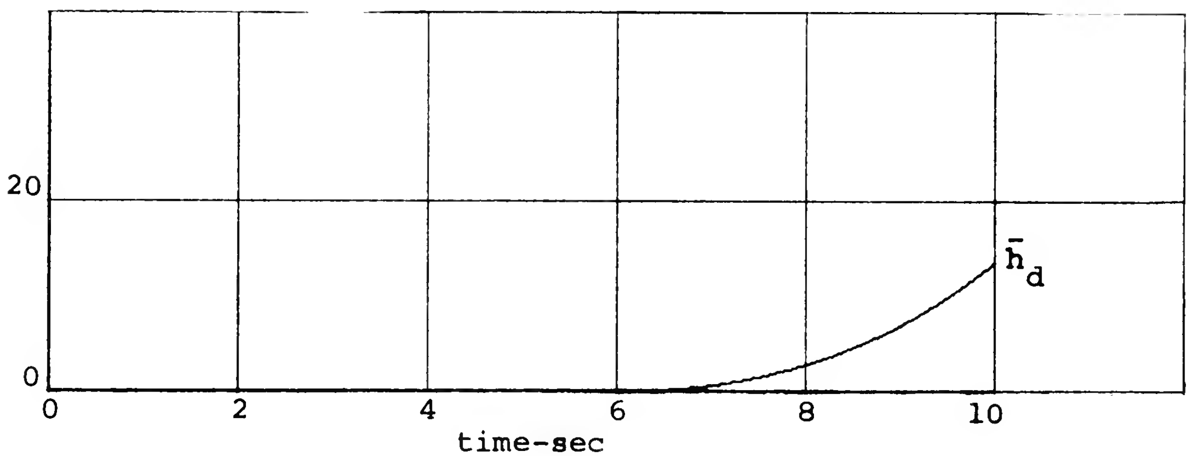

Figure 8

Desired Revised Altitude Trajectory

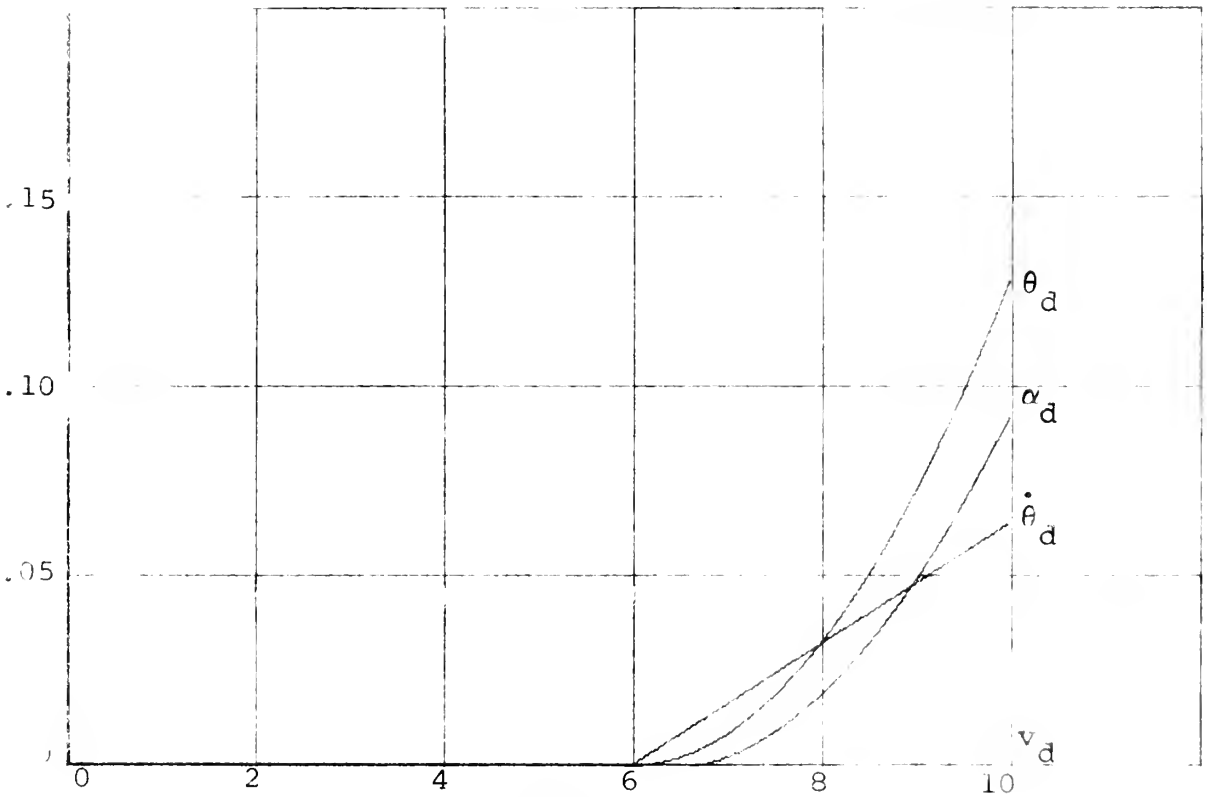

Figure 9

Desired Velocity, Angle of Attack, Pitch Angle, and Pitch Rate Trajectory

C. PERFORMANCE MEASURE

The mathematical form of the performance measure introduced in Chapter IV was

$$J = 1/2 \; \| \underline{x}(t_f) - \underline{r}(t_f) \|_H^2$$

$$+ \frac{1}{2} \int_{t_o}^{t_f} \left[\| \underline{x}(t) - \underline{r}(t) \|_{Q(t)}^2 + \| \underline{u}(t) \|_{R(t)}^2 \right] dt \tag{5.27}$$

One of the advantages of this quadratic performance measure is that the elements of the H, Q(t) and R(t) matrices,which shall be called the weighting matrices, can be related to the design parameters of the system and chosen to satisfy the design objectives. If these weighting matrices meet the requirements established in Chapter IV, specifically H and Q(t) are positive semi-definite and R(t) is positive definite, the optimal control law can be found, and is unique for the selected set of weighting matrices.

Previous investigations dealing with similar airplane landing problems (Refs. 8 and 9) have indicated that the weighting matrices must be diagonal matrices with positive diagonal entries. In other words, the weighting matrices must have non-zero weighting values specified for each state and control. As a result, in the initial trials it was assumed that the weighting matrices are diagonal and time invariant over the interval of interest. Since the form of the weighting matrices was so selected, each diagonal element of the respective matrix could be related to a specific state variable or control, and therefore, theoretically, could be assigned a proper value to meet established specifications. Unfortunately, this selection of weighting factors was not simply accomplished. In fact, it became obvious during the investigation that the state trajectory was very sensitive to changes in the weighting matrices. (See related discussion in Chapter VII.)

The selection of values for the weighting matrices was carried out by a trial-and-error method wherein a set of weighting matrices was selected, the optimal control law computed, the optimal trajectory calculated, and the results compared with established specifications. In effect, the final weighting matrix selection was based on obtaining a realistic optimal trajectory which conforms to the specifications.

Appendix B contains the numerical values used in the weighting matrices for this investigation.

VI. INVESTIGATION PROCEDURE

Since the previous discussion has presented the procedures used in modeling the plant, developing the desired trajectory, and related topics, only the procedure used to obtain the optimal control will be presented. The actual cases investigated will also be discussed in the last section of this chapter.

A. PROCEDURE

The following procedure was used to obtain the optimal control:

1. A set of representative values were selected for the diagonal elements of the weighting matrices (H, Q, and R) of the performance measure, equation (5.27).

2. Equations (4.6) and (4.8) were simultaneously integrated from $t = t_f$ to $t = 0$ to obtain values for K(t) and $\underline{s}(t)$ respectively.

3. Using the results of step 2, equations (4.4) and (4.5) were solved to obtain values for F(t) and $\underline{g}(t)$ respectively.

4. The state equations (3.11) were integrated from $t = 0$ to $t = t_f$, using the results of step 3 to obtain $\underline{u}^*(t)$ of equation (4.22).

5. The optimal control and optimal trajectory obtained from step 4 were observed to ascertain compliance

with problem specifications. If the results were not admissible or unrealistic, the values of the elements of the weighting matrices were changed and steps 2 through 4 were repeated.

To implement the foregoing procedure, a computer program was written in Fortran IV for use in the IBM System/360 digital computer. System/360 Scientific Subroutines were used in the program to accomplish the matrix algebra. An existing Fourth-Order Runge-Kutta subroutine developed at the Naval Postgraduate School was used for the required numerical integration. The computer program, including the integration subroutine, is presented after the appendices.

B. CASES INVESTIGATED

Two cases were investigated in the initial study. Case I assumes that the velocity of the airplane is constant and elevator deflection is the only control available; Case II considers the plant and controls as previously formulated. As a result, two different state models were used.

1. Case I

Since it is assumed that the velocity is constant and that elevator deflection is the only control available, the state model, equation (3.11) reduces to

$$\underline{\dot{x}}(t) = \begin{pmatrix} a_{22} & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ a_{42} & 0 & a_{44} & 0 \\ a_{52} & a_{53} & 0 & 0 \end{pmatrix} \underline{x}(t)$$

$$+ \begin{pmatrix} b_{21} \\ 0 \\ b_{41} \\ 0 \end{pmatrix} \underline{u}(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ c_{51} \end{pmatrix} \tag{6.1}$$

where

$$\underline{x} = \begin{pmatrix} \alpha \\ \theta \\ \dot{\theta} \\ h \end{pmatrix} \tag{6.2}$$

and

$$\underline{u} = \delta_e \tag{6.3}$$

By the same argument as was presented in Chapter IV, this state equation can be manipulated into the revised state equation

$$\dot{\underline{\bar{x}}}(t) = A\underline{\bar{x}}(t) + B\underline{u}(t) \tag{6.4}$$

where

$$\underline{\bar{x}} = \begin{pmatrix} \alpha \\ \theta \\ \dot{\theta} \\ \bar{h} \end{pmatrix} \tag{6.5}$$

and the elements of A and B remain identical to those of equation (6.1).

2. <u>Case II</u>

The state model, equation (3.11), and the revised state model, equation (4.20), as previously presented, were used in Case II.

In order to evaluate the optimal trajectory, three different sets of initial conditions were selected. The first assumes that the airplane is in an ideal flight condition, or, in terms of the revised state model, all initial state values are zero. The other two sets were selected to conform to the worst initial flight conditions that could be reasonably expected. The following listing discusses these initial flight conditions.

a. HIGH AND FAST. This set of initial conditions assumes that the airplane was at the upper altitude limit of the ILS Category II "window" and that the airplane velocity is above v_o, the equilibrium airspeed. In Case I, the second assumption is not, strictly speaking, applicable since it is assumed that velocity was constant. However, the condition was simulated by assuming that the initial angle of attack, $\alpha(t_o)$, and pitch angle, $\theta(t_o)$, were below their respective equilibrium values. This condition would result in actual flight if the airplane velocity were above the equilibrium flight velocity. It was further assumed that the initial pitch angle rate, $\dot{\theta}(t_o)$, was zero.

b. LOW AND SLOW. This set of initial conditions assumes that the airplane is at the lower altitude limit of the ILS Category II "window" and that the airplane velocity is below the equilibrium airspeed. By the same reasoning as before, the second assumption was simulated for Case I by assuming that the initial angle of attack and pitch angle were above their respective equilibrium values. Again, the initial pitch angle rate was assumed to be zero.

Table B presents a summary of the cases investigated and includes the initial values used for each case.

Table B

Summary of Cases Investigated

Case	Initial Flight Condition	Applicable State Equations	Initial State Conditions
IA	IDEAL	(6.1,6.4)	$(0,0,0,0)^T$
IB	HIGH AND FAST	(6.1,6.4)	$(-0.03,-0.03,0,112.0)^T$
IC	LOW AND SLOW	(6.1,6.4)	(0.03,0.03,0,88.0)
IIA	IDEAL	(3.11,4.20)	$(0,0,0,0,0)^T$
IIB	HIGH AND FAST	(3.11,4.20)	$(5.0,-0.03,-0.03,0,112.0)^T$
IIC	LOW AND SLOW	(3.11,4.20)	$(-5.0,0.03,0.03,0,88.0)^T$

VII. RESULTS

A general result is first presented, followed by specific results, in graphical form, for each of the cases investigated. The final section of this chapter discusses a problem encountered during the investigation.

A. GENERAL RESULTS

The optimal control for an all-weather landing in the F-4J airplane was derived (see Specific Results below) using the techniques presented. Since some simplifying assumptions were made to reduce the complexity of the problem, the results are not conclusive, but serve to demonstrate that the design of an automatic controller for the landing of an airplane is feasible by formulating the problem as a tracking problem and applying optimal control theory. By systematically eliminating some of the assumptions made in this study, a closer approximation to the actual airplane landing problem can be accomplished.

Actual implementation of an automatic controller to provide the optimal control derived by these techniques would require that a digital computer be placed aboard the airplane which may impose an undersirable penalty. If so, the optimal control still provides the goal for the design of any sub-optimal controller and hence is of great value to the control engineer.

B. SPECIFIC RESULTS

Specific results for each of the cases investigated are presented in graphical form and are summarized below.

1. The solution to equation (4.6), the K(t) values, are presented for:

   a. Case I in Figure 10

   b. Case II in Figure 21

2. The solution to equation (4.8), the $\underline{s}(t)$ values, are presented for:

   a. Case I in Figure 11

   b. Case II in Figure 22

3. The optimal control is depicted for:

   a. Case IA in Figure 12

   b. Case IB in Figure 15

   c. Case IC in Figure 18

   d. Case IIA in Figure 23

   e. Case IIB in Figure 26

   f. Case IIC in Figure 29

4. The optimal and desired altitude trajectories are presented for:

   a. Case IA in Figure 13

   b. Case IB in Figure 16

   c. Case IC in Figure 19

   d. Case IIA in Figure 24

   e. Case IIB in Figure 27

   f. Case IIC in Figure 30

5. The components of the optimal trajectory, v, α, θ, and $\dot{\theta}$, together with the related airplane parameters, rate of descent and normal acceleration, are presented for:

a. Case IA in Figure 14

b. Case IB in Figure 17

c. Case IC in Figure 20

d. Case IIA in Figure 25

e. Case IIB in Figure 28

f. Case IIC in Figure 31

In all cases, the results were within established specification limits and were considered realistic for an airplane landing. The anticipatory nature of the optimal control was evident in the results, especially in Case IIA, where the optimal control produced a finite thrust perturbation at the initial time, although the airplane was assumed to be in an ideal flight condition. This apparent lack of continuity, along with the discontinuity encountered in the optimal control of Cases IB, IC, IIB, and IIC at t = 0, was predictable, since it was assumed that the controls were instantaneously available. Development of a state model to include control dynamics, presented in Chapter III, will presumably eliminate this apparent discrepancy.

c. PROBLEM AREA

The importance of proper selection of the constant values for the elements of the diagonal weighting matrices of the performance measure is evident; namely, no unique values exist and hence proper selection is based on producing an admissible control and trajectory when the optimization technique is applied. In this investigation, where many specifications had to be simultaneously satisfied, this selection became paramount, especially when it became apparent that several components of the state trajectory were sensitive to changes in one element of the weighting matrices. As a result, the trial-and-error process required to obtain values for these weighting matrices was tedious and time-consuming. Further study is warranted toward the development of a scheme or method to aid the control engineer in the selection of these weighting matrices.

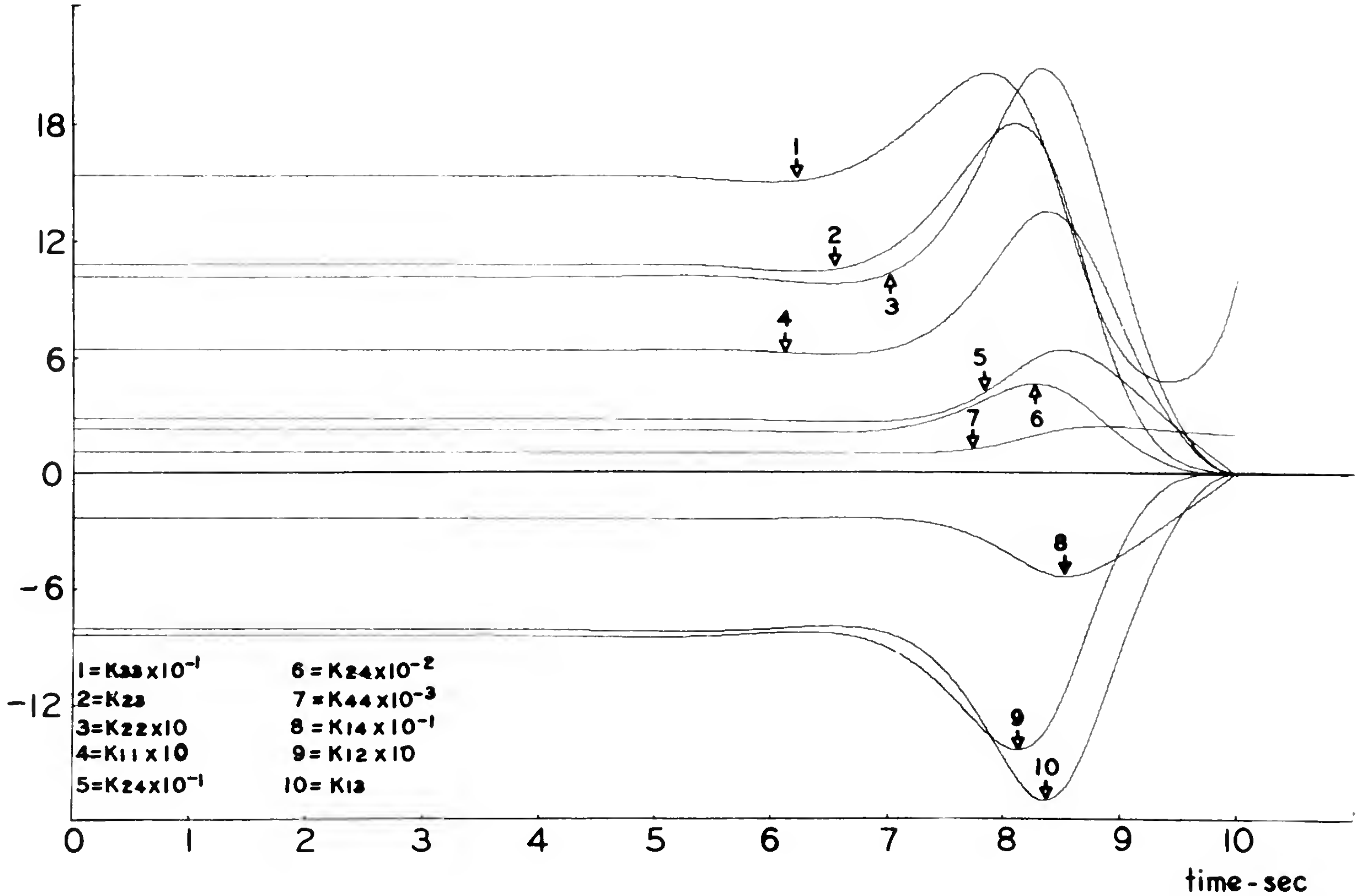

FIGURE 10
THE SOLUTION OF THE
RICCATI EQUATION FOR CASE I

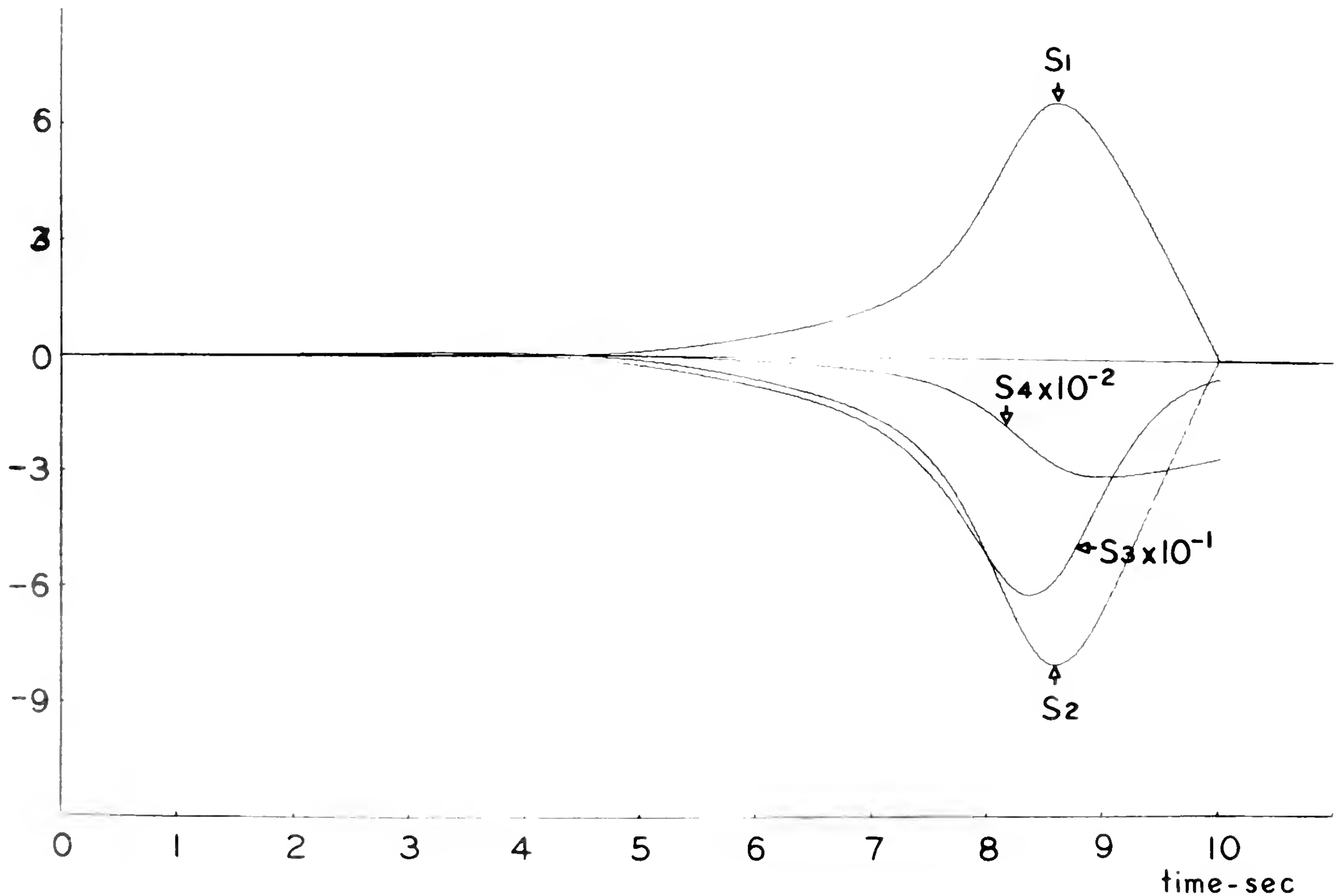

FIGURE 11
THE AUXILIARY
FUNCTION $\underline{s}$ FOR CASE I

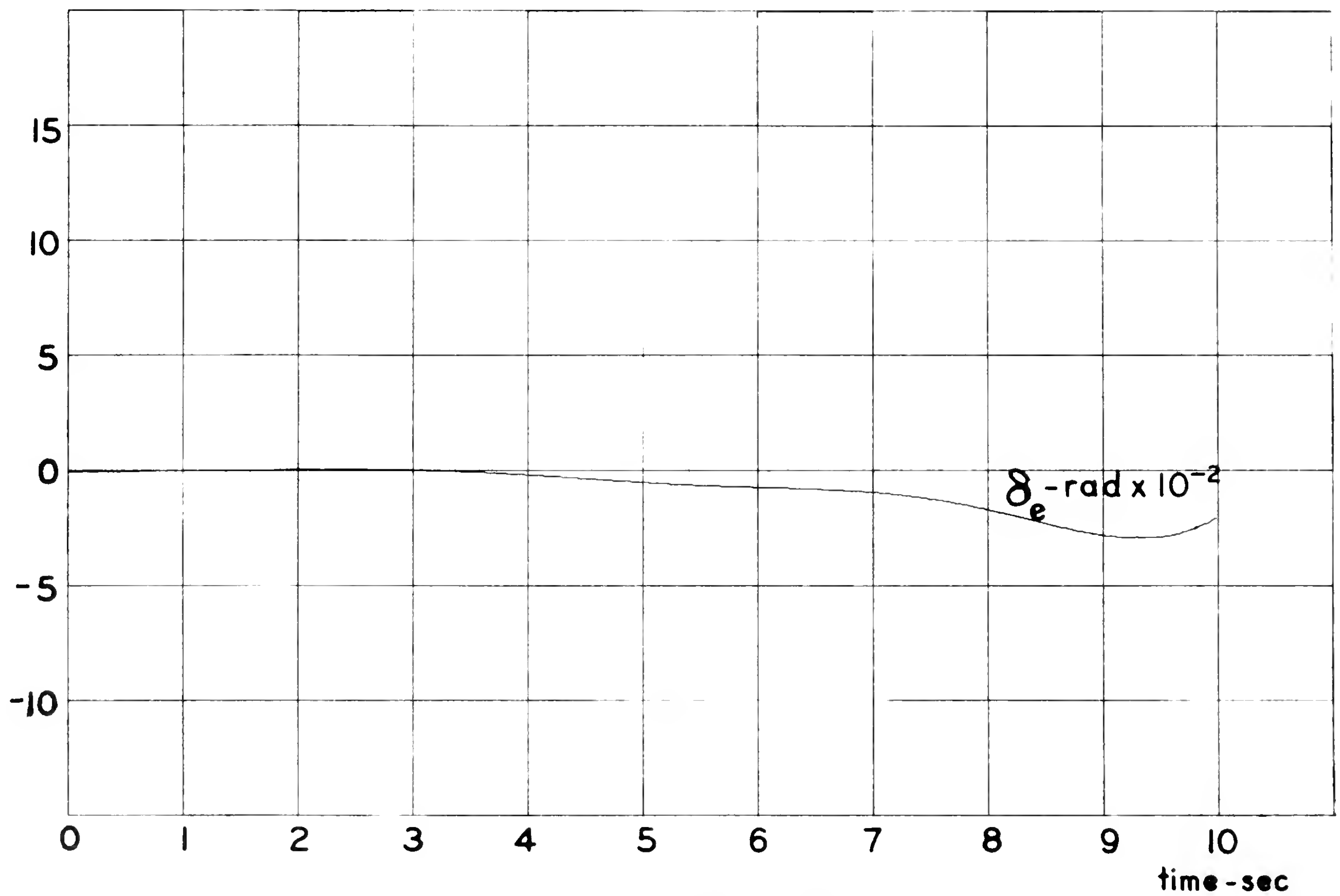

FIGURE 12
THE OPTIMAL
CONTROL FOR CASE IA

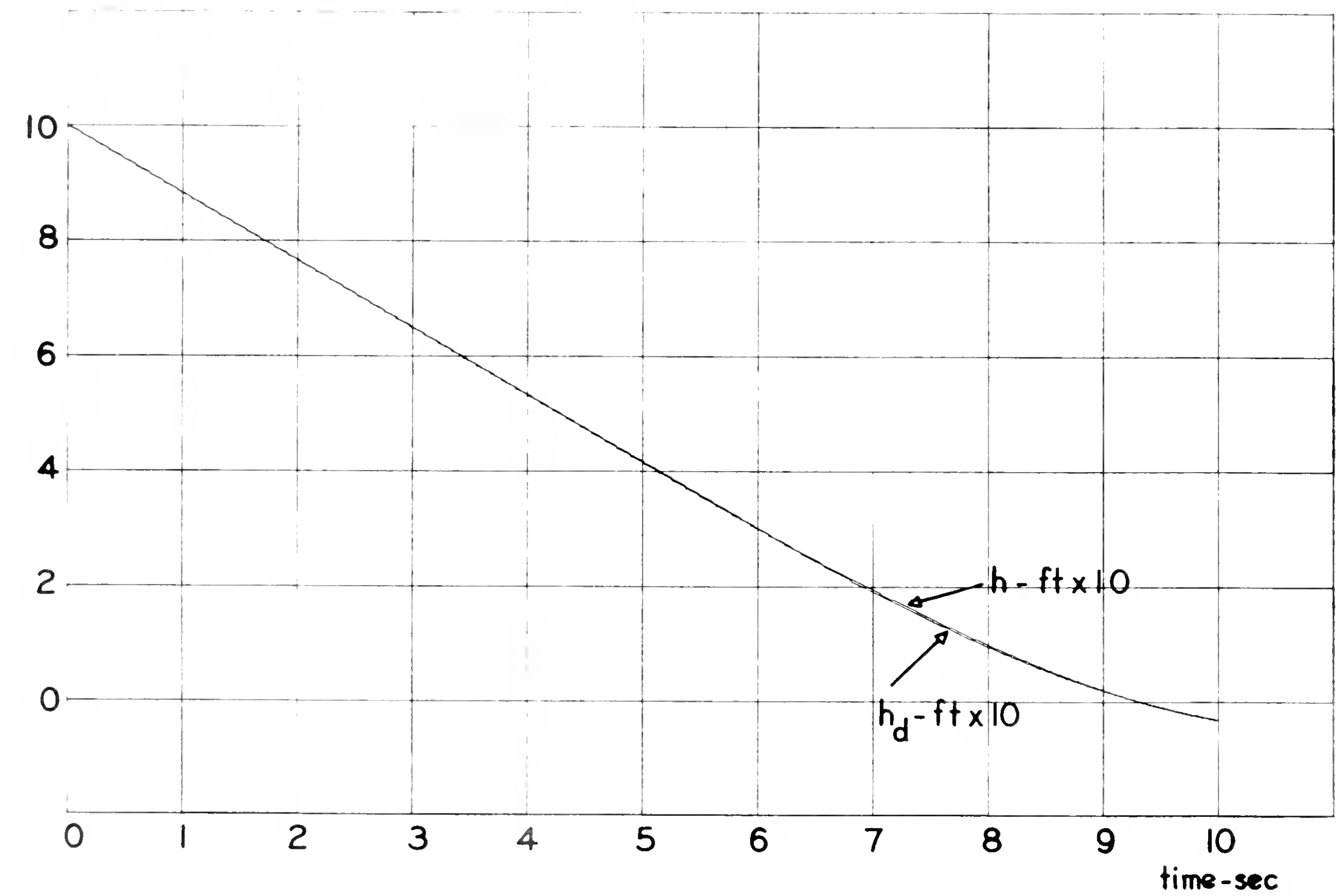

FIGURE 13
THE OPTIMAL AND DESIRED
ALTITUDE TRAJECTORIES FOR CASE IA

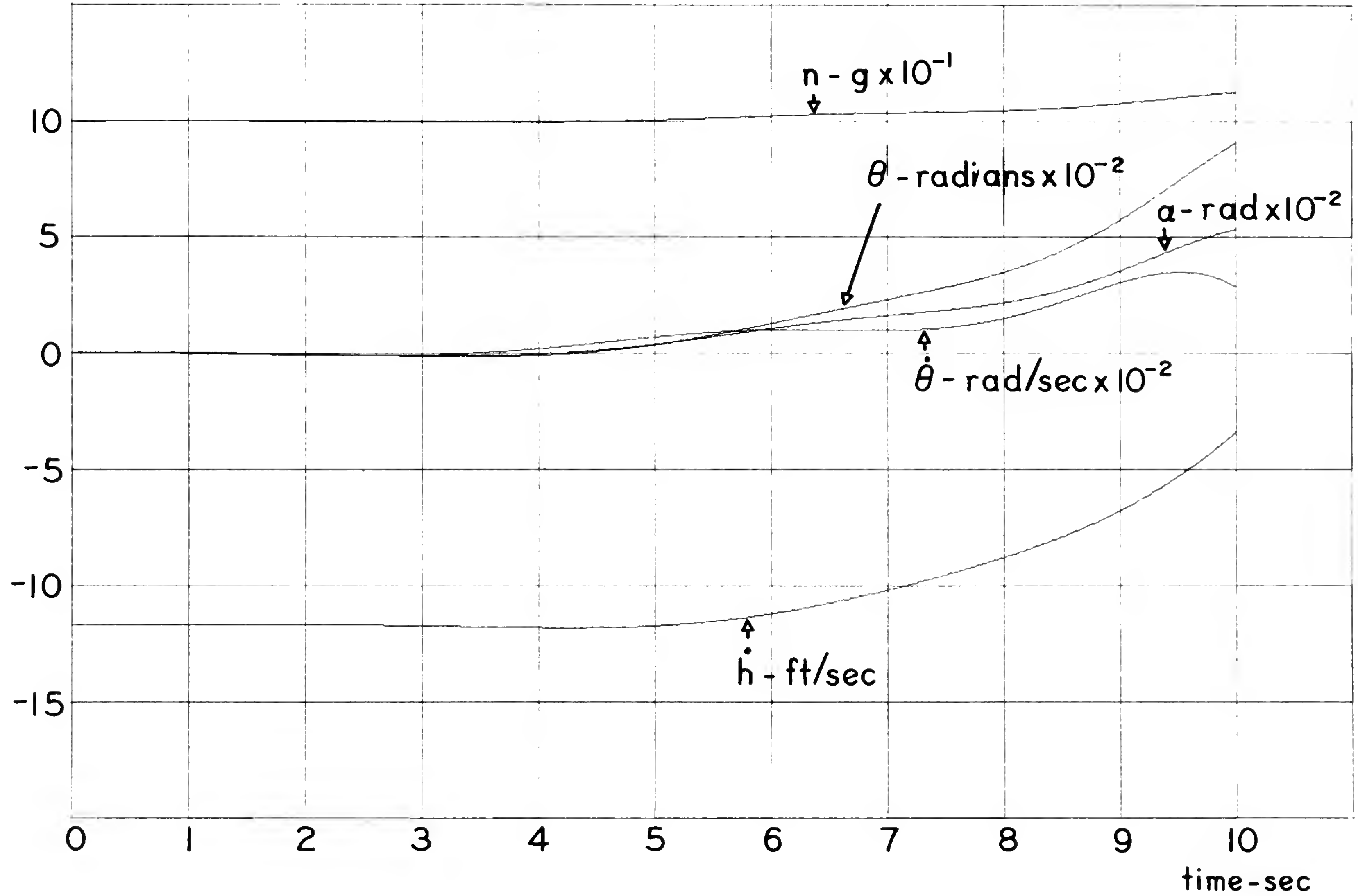

FIGURE 14
THE OPTIMAL α, θ, AND $\dot{\theta}$
HISTORIES AND h, $\dot{h}$ TRACES FOR CASE IA

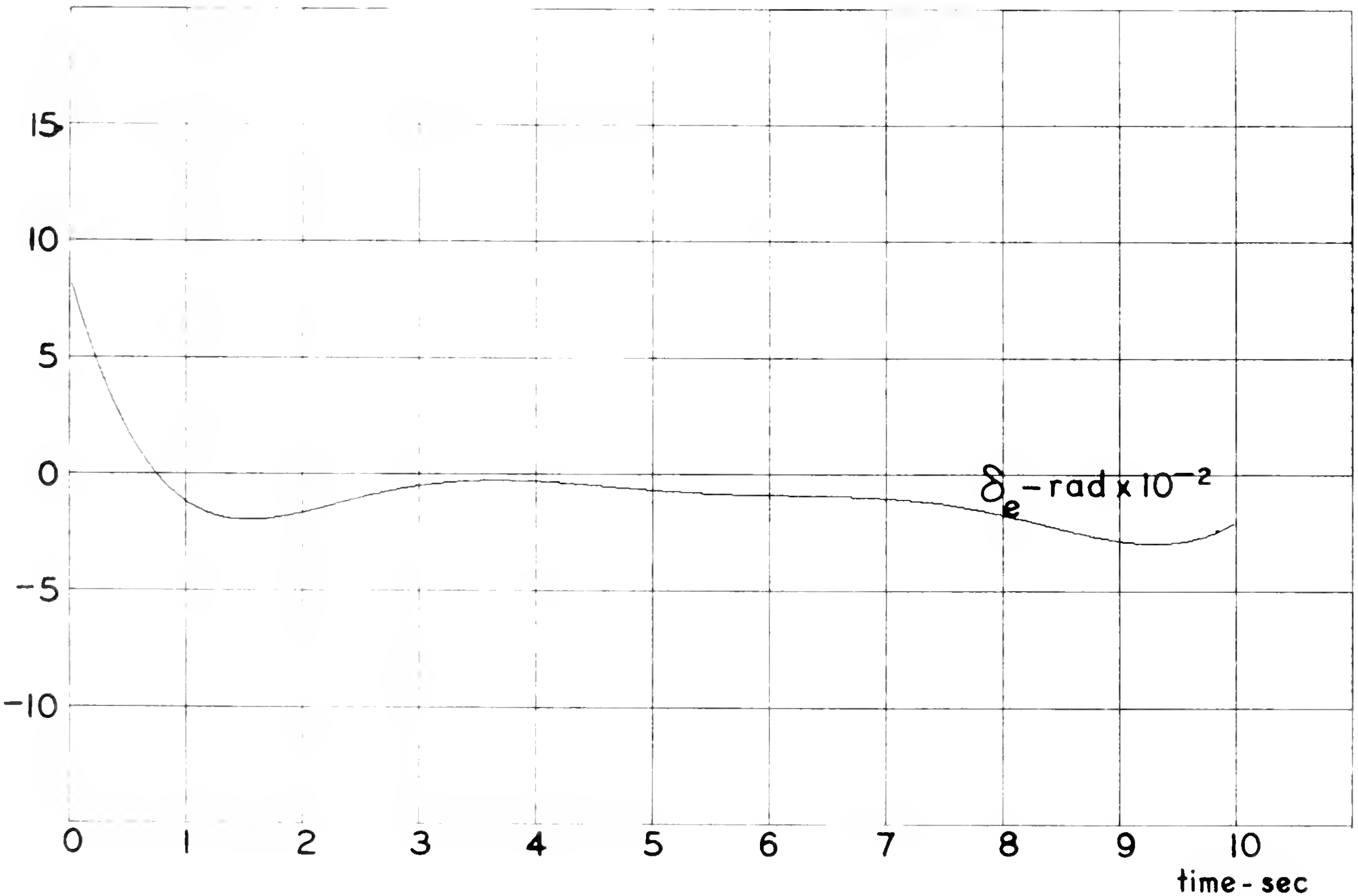

FIGURE 15
THE OPTIMAL
CONTROL FOR CASE IB

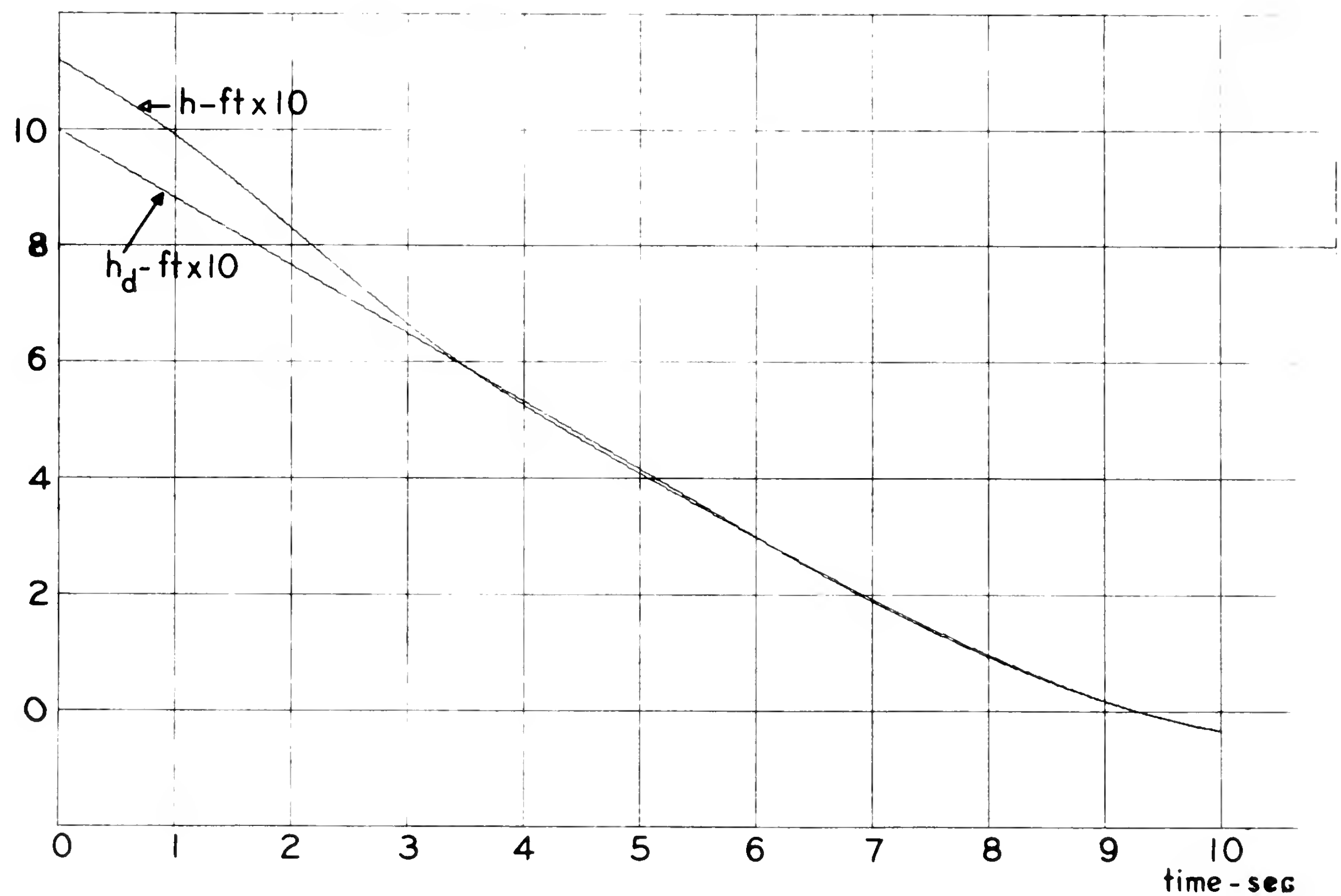

FIGURE 16
THE OPTIMAL AND DESIRED
ALTITUDE TRAJECTORIES FOR CASE IB

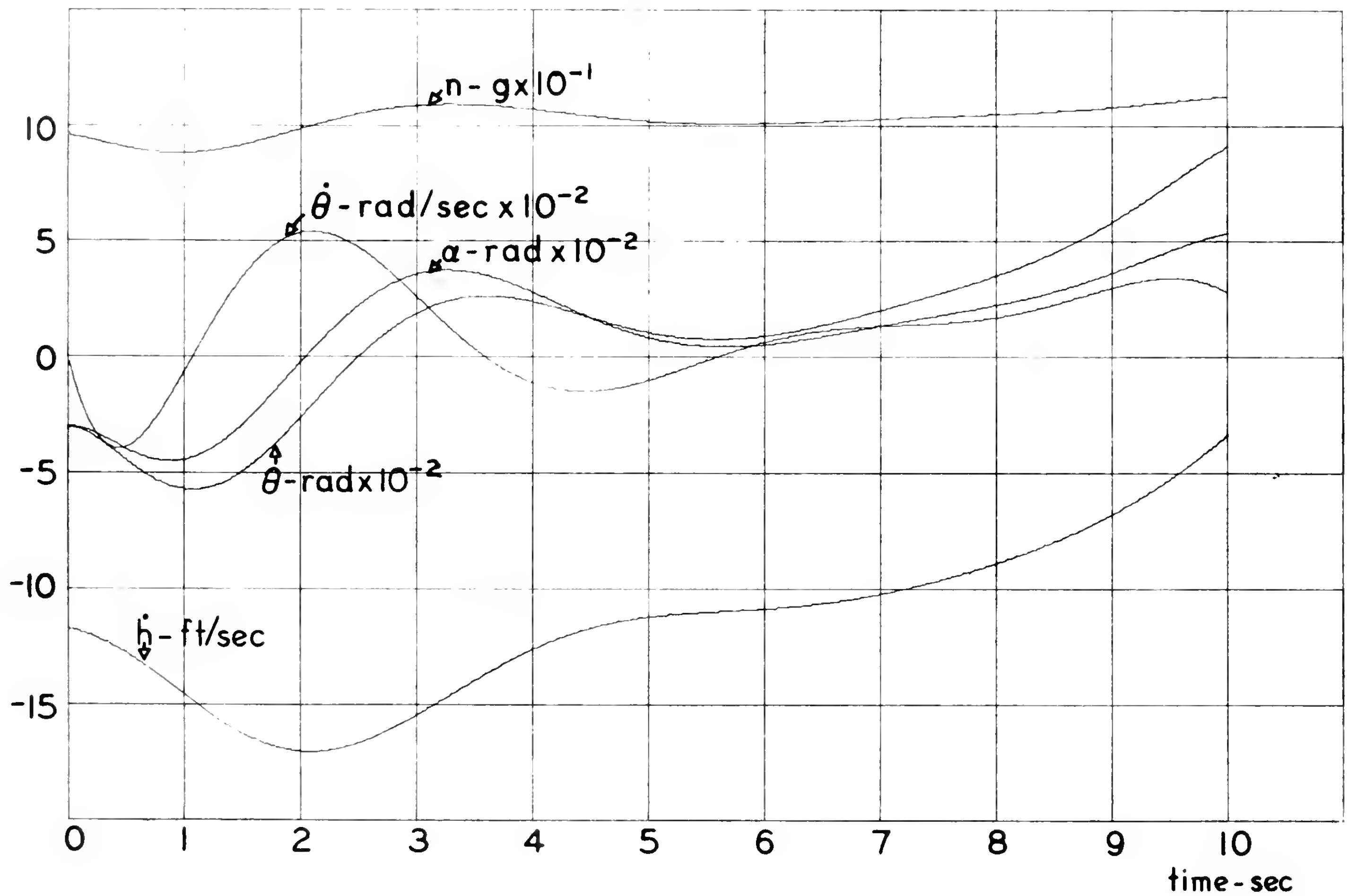

FIGURE 17
THE OPTIMAL α, θ, AND $\dot{\theta}$
HISTORIES AND n, $\dot{h}$ TRACES FOR CASE IB

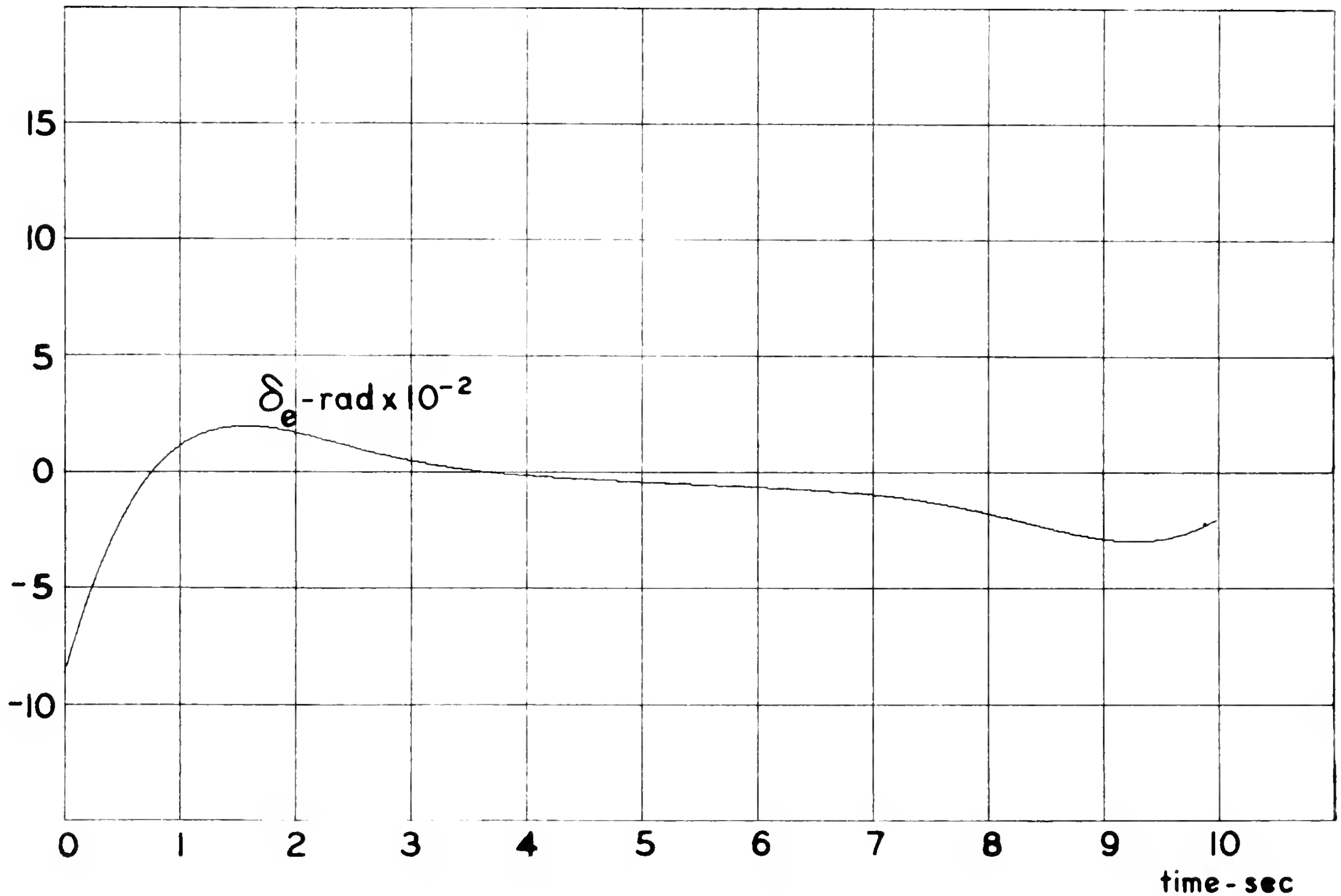

FIGURE 18
THE OPTIMAL
CONTROL FOR CASE IC

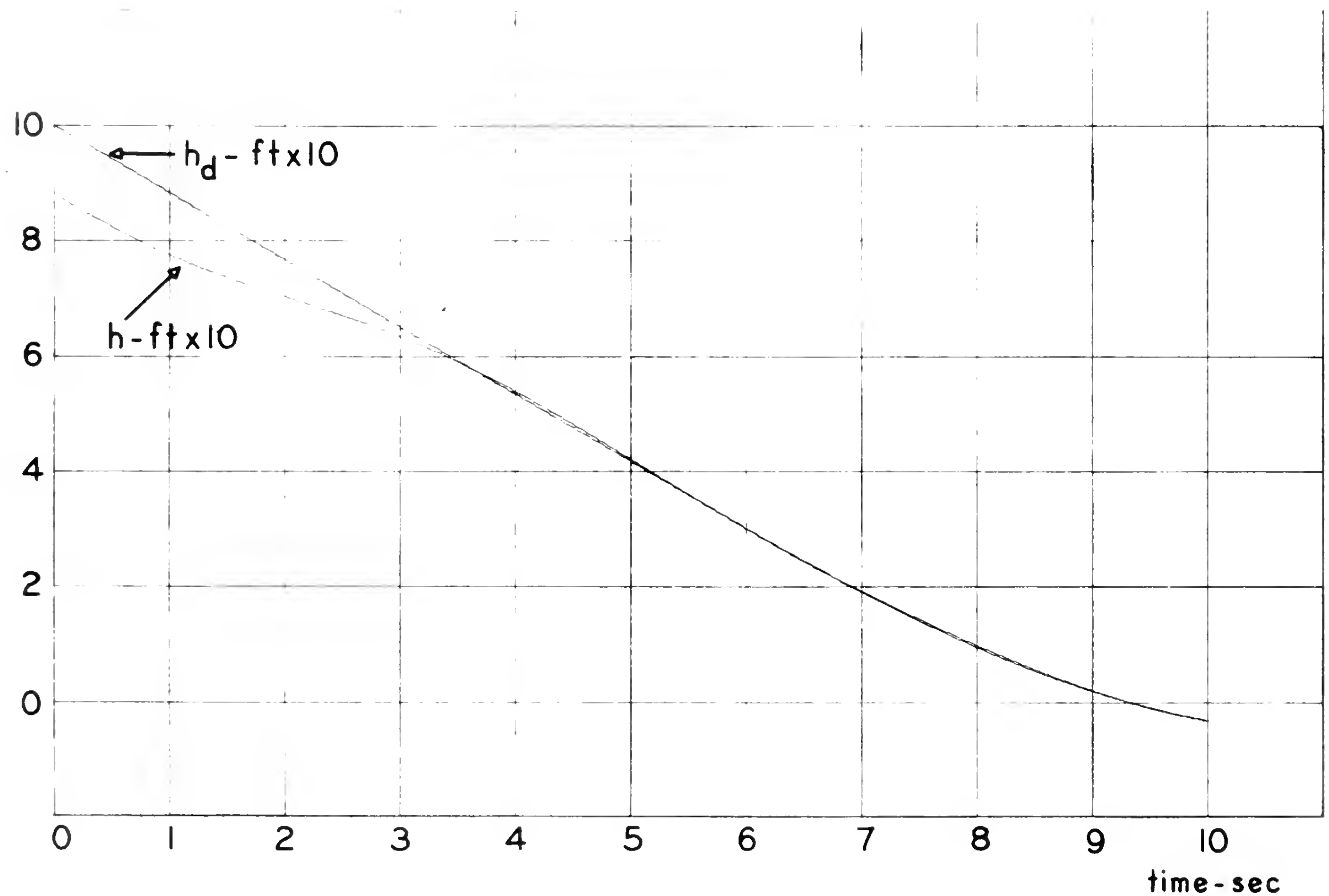

FIGURE 19
THE OPTIMAL AND DESIRED
ALTITUDE TRAJECTORIES FOR CASE IC

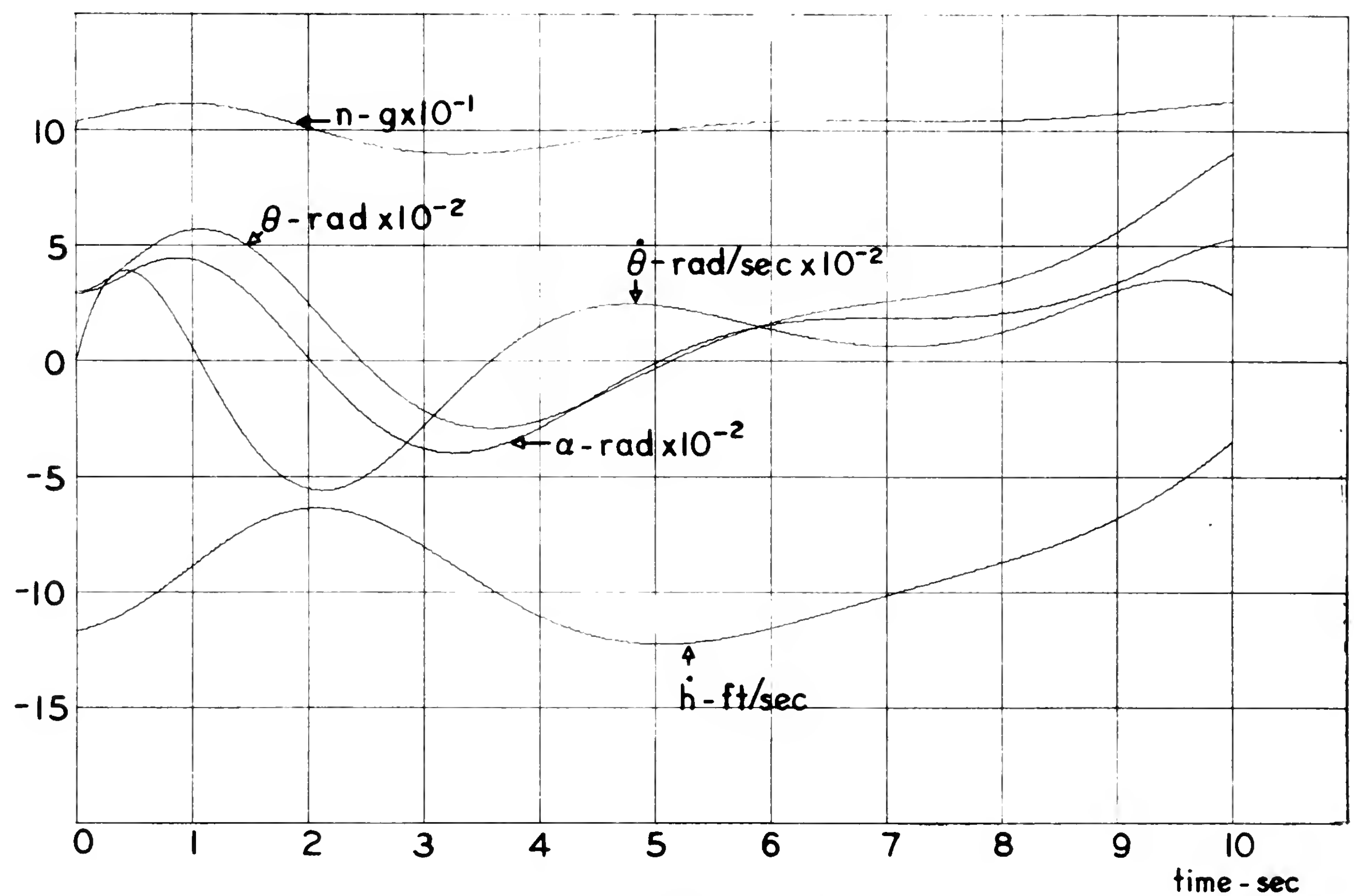

FIGURE 20
THE OPTIMAL α, θ, AND $\dot{\theta}$ HISTORIES AND n, $\dot{h}$ TRACES FOR CASE IC

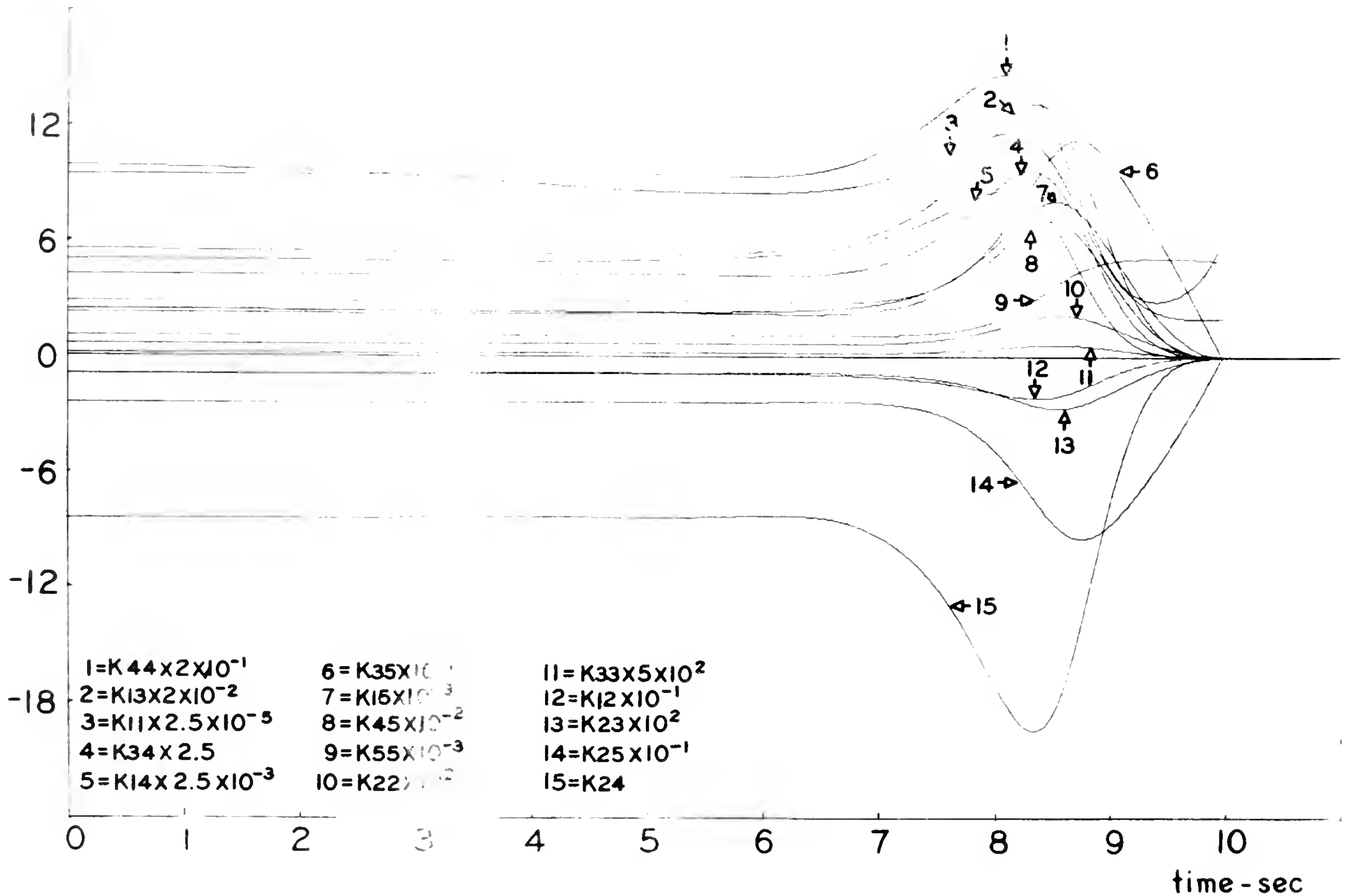

FIGURE 21
THE SOLUTION OF THE
RICCATI EQUATION FOR CASE II

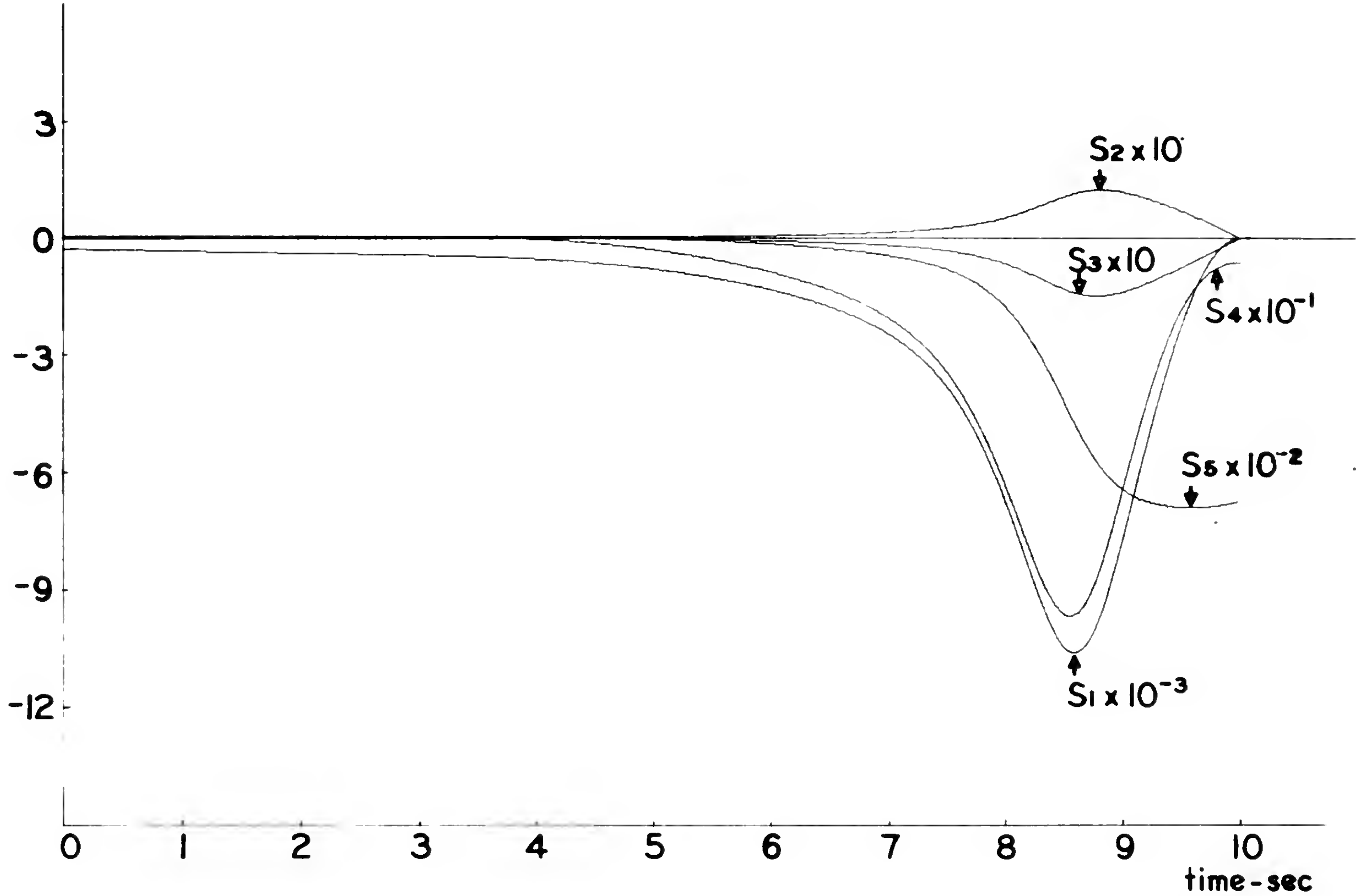

FIGURE 22
THE AUXILIARY
FUNCTION $\underline{s}$ FOR CASE II

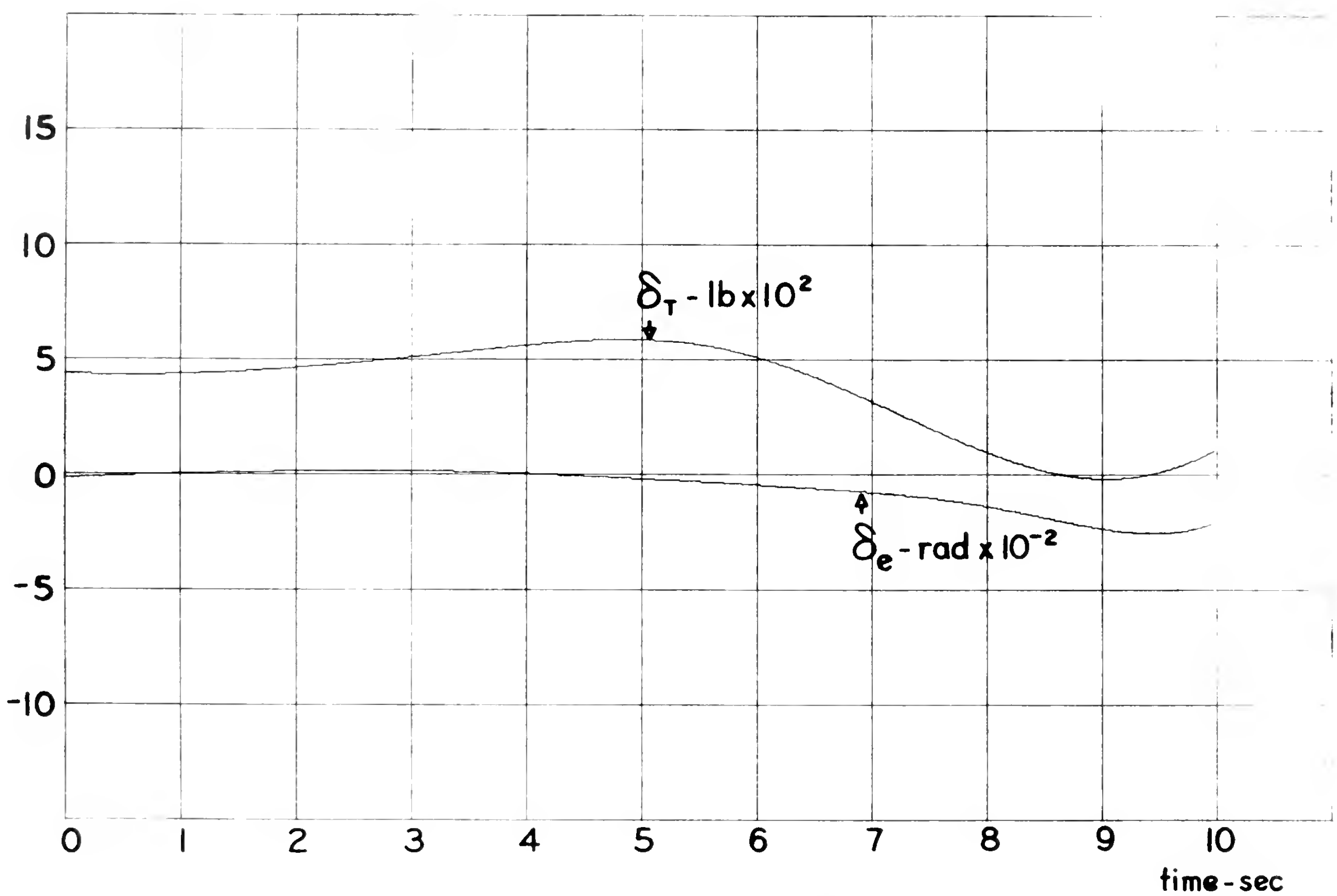

FIGURE 23
THE OPTIMAL
CONTROL FOR CASE IIA

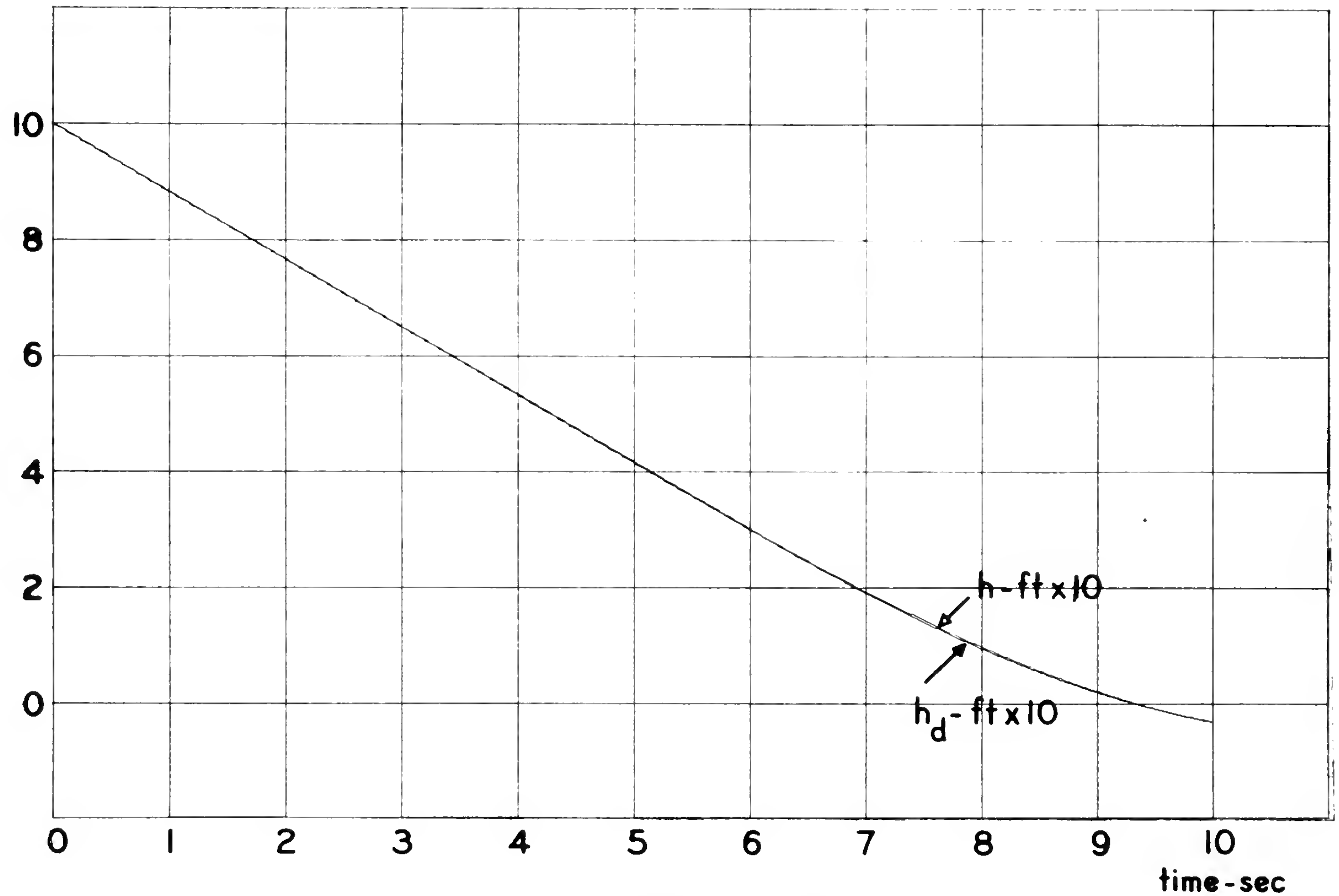

FIGURE 24
THE OPTIMAL AND DESIRED
ALTITUDE TRAJECTORIES FOR CASE IIA

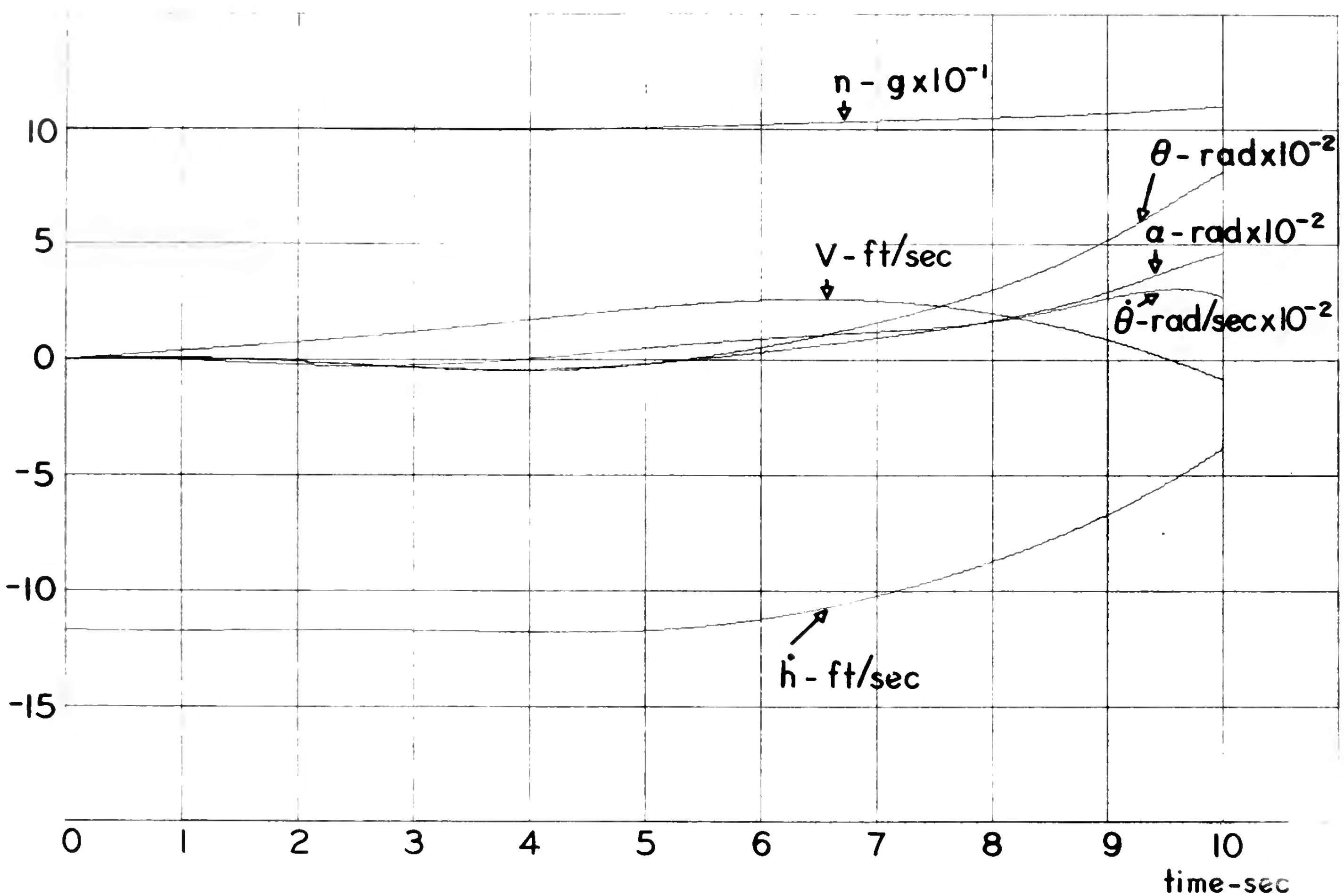

FIGURE 25

THE OPTIMAL v, α, θ, AND $\dot{\theta}$ HISTORIES AND n, $\dot{h}$ TRACES FOR CASE IIA

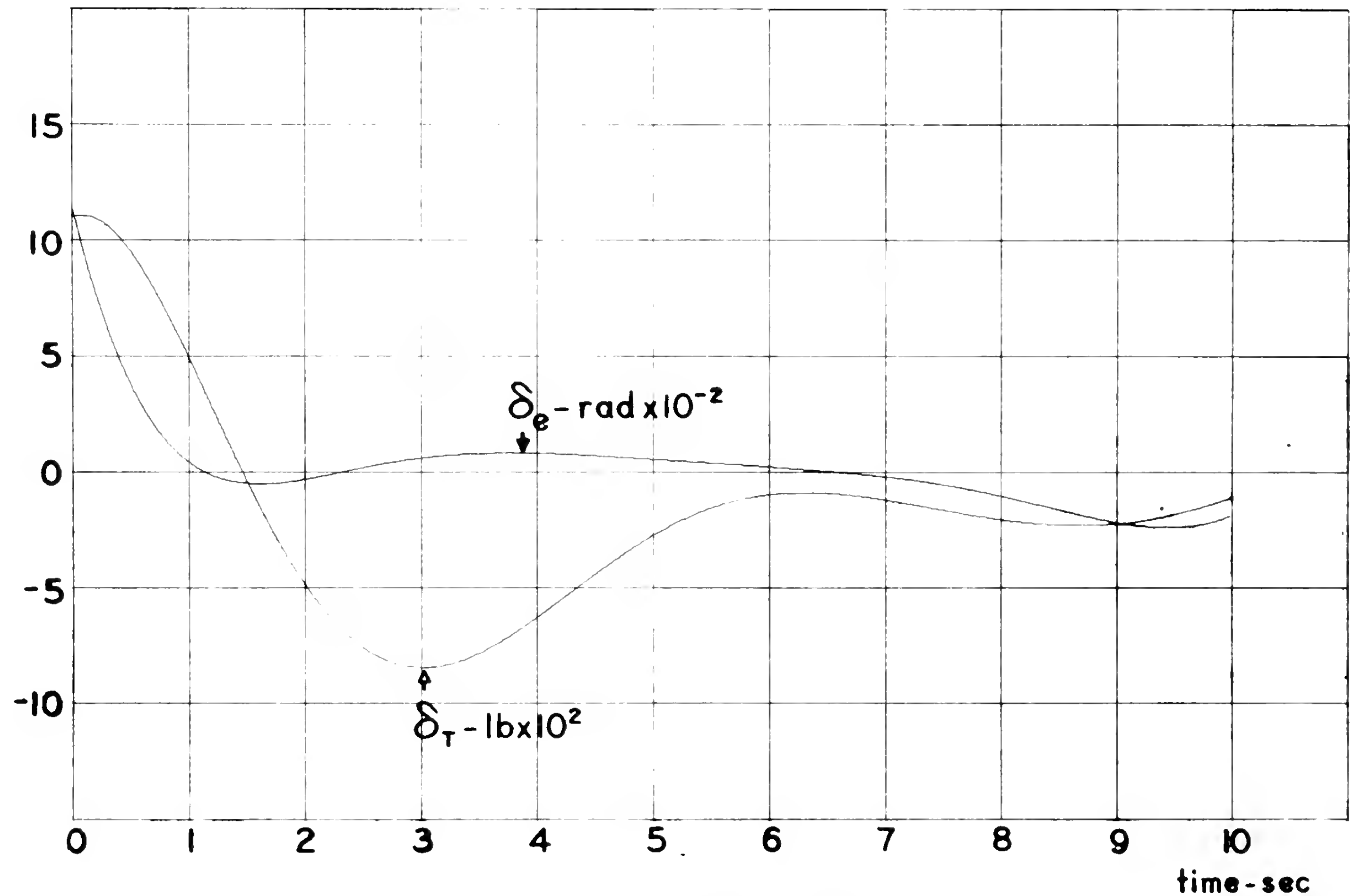

FIGURE 26
THE OPTIMAL
CONTROL FOR CASE IIB

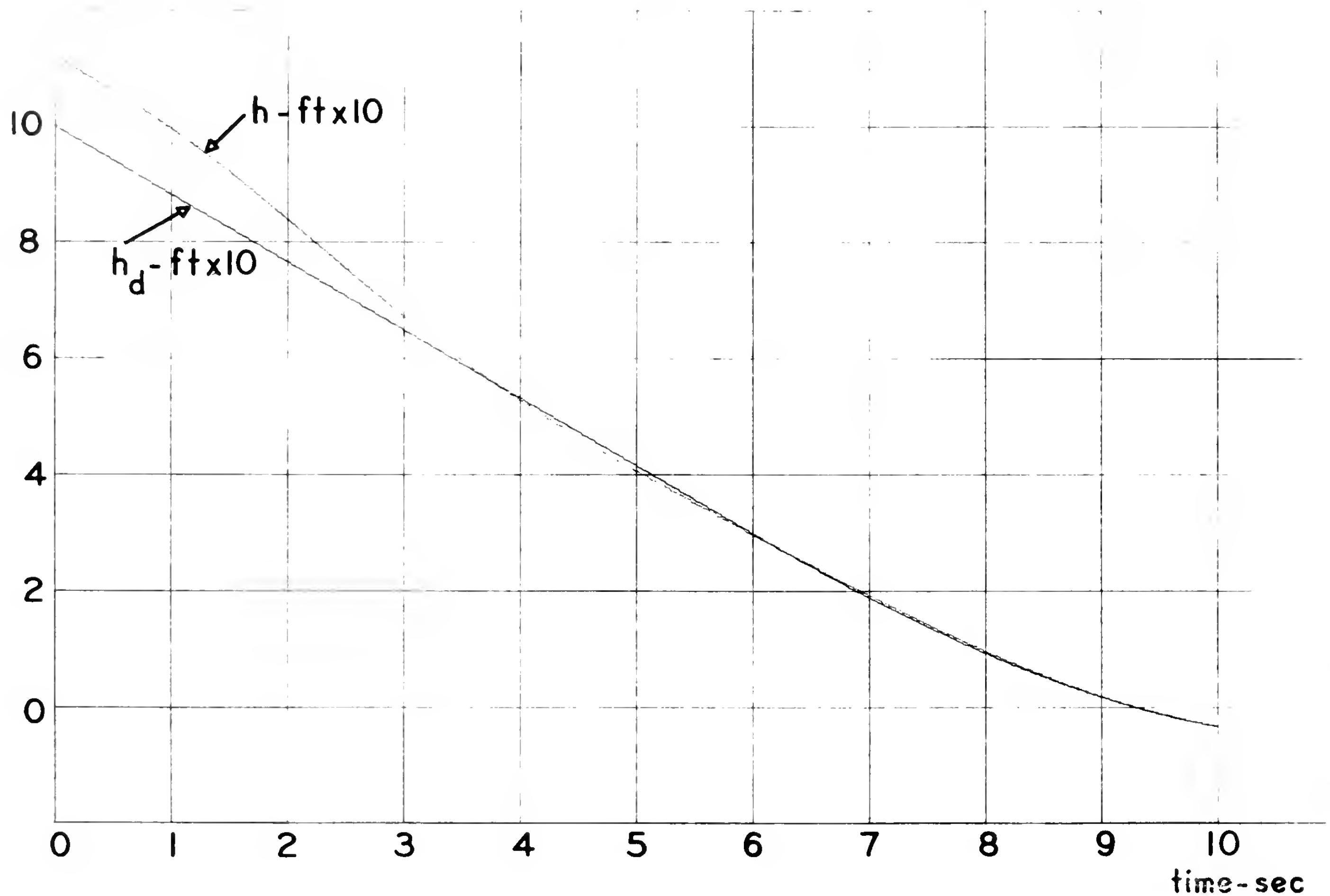

FIGURE 27
THE OPTIMAL AND DESIRED
ALTITUDE TRAJECTORIES FOR CASE IIB

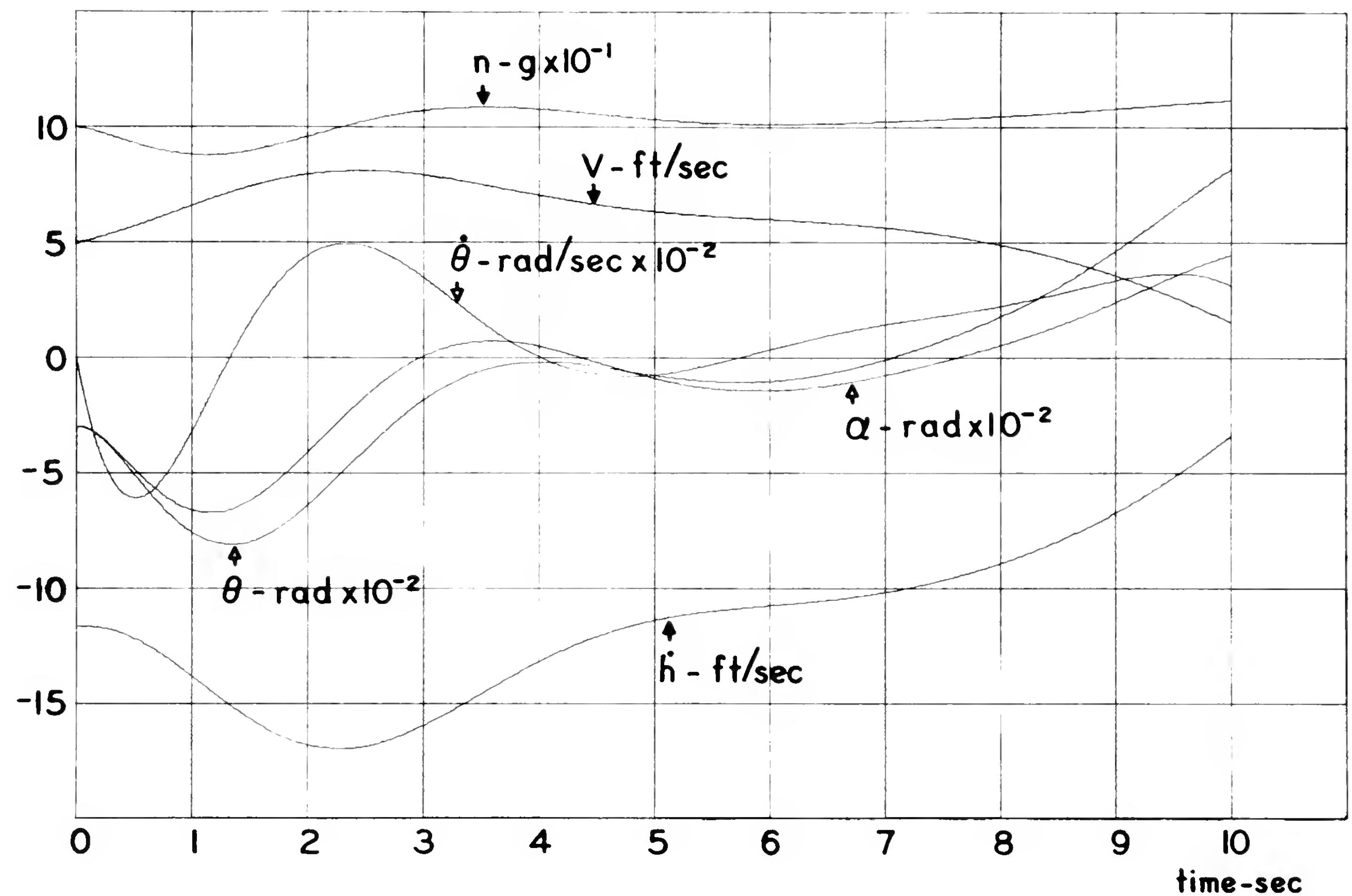

FIGURE 28
THE OPTIMAL v, α, θ, AND $\dot{\theta}$
HISTORIES AND n, $\dot{h}$ TRACES FOR CASE IIB

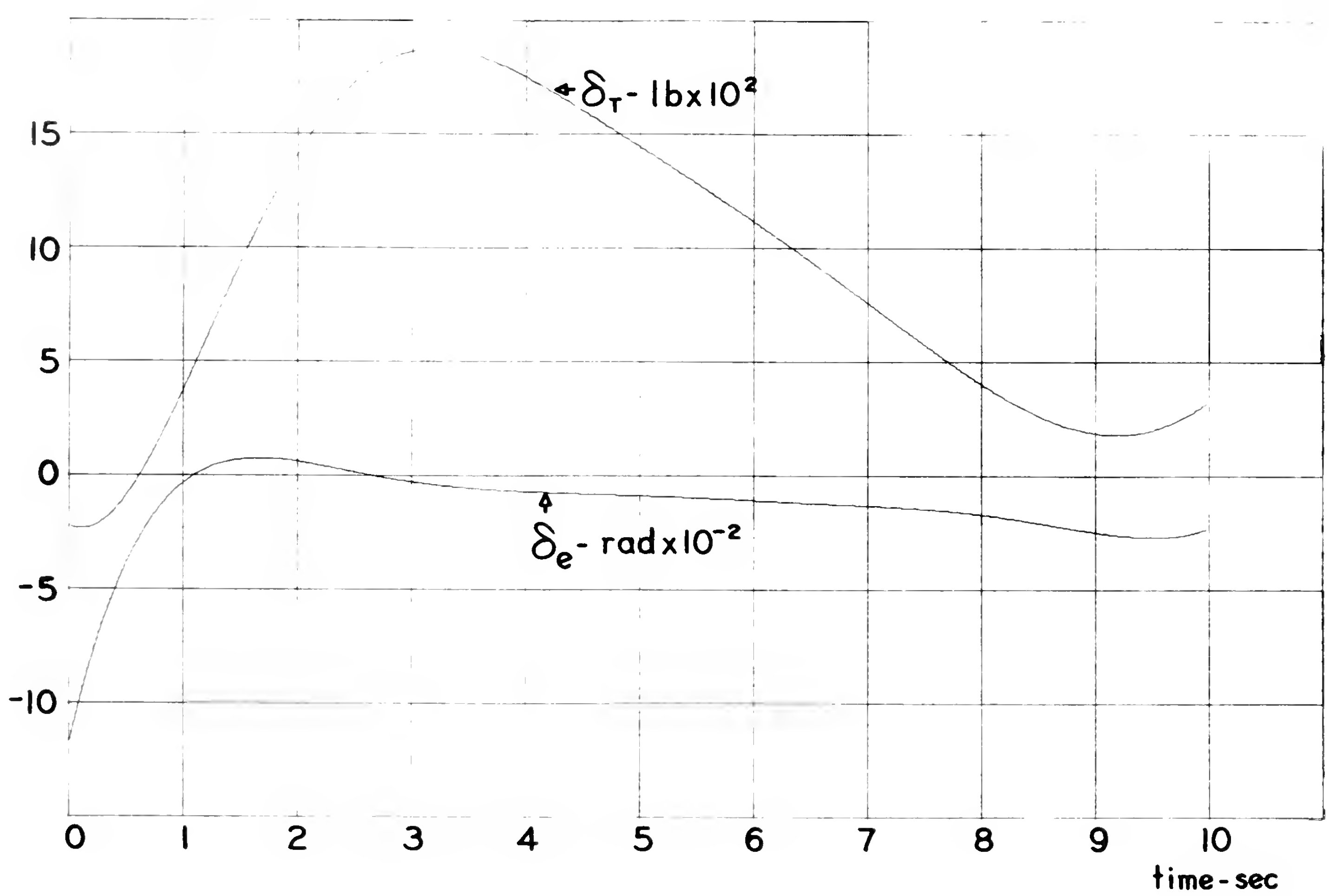

FIGURE 29
THE OPTIMAL
CONTROL FOR CASE IIC

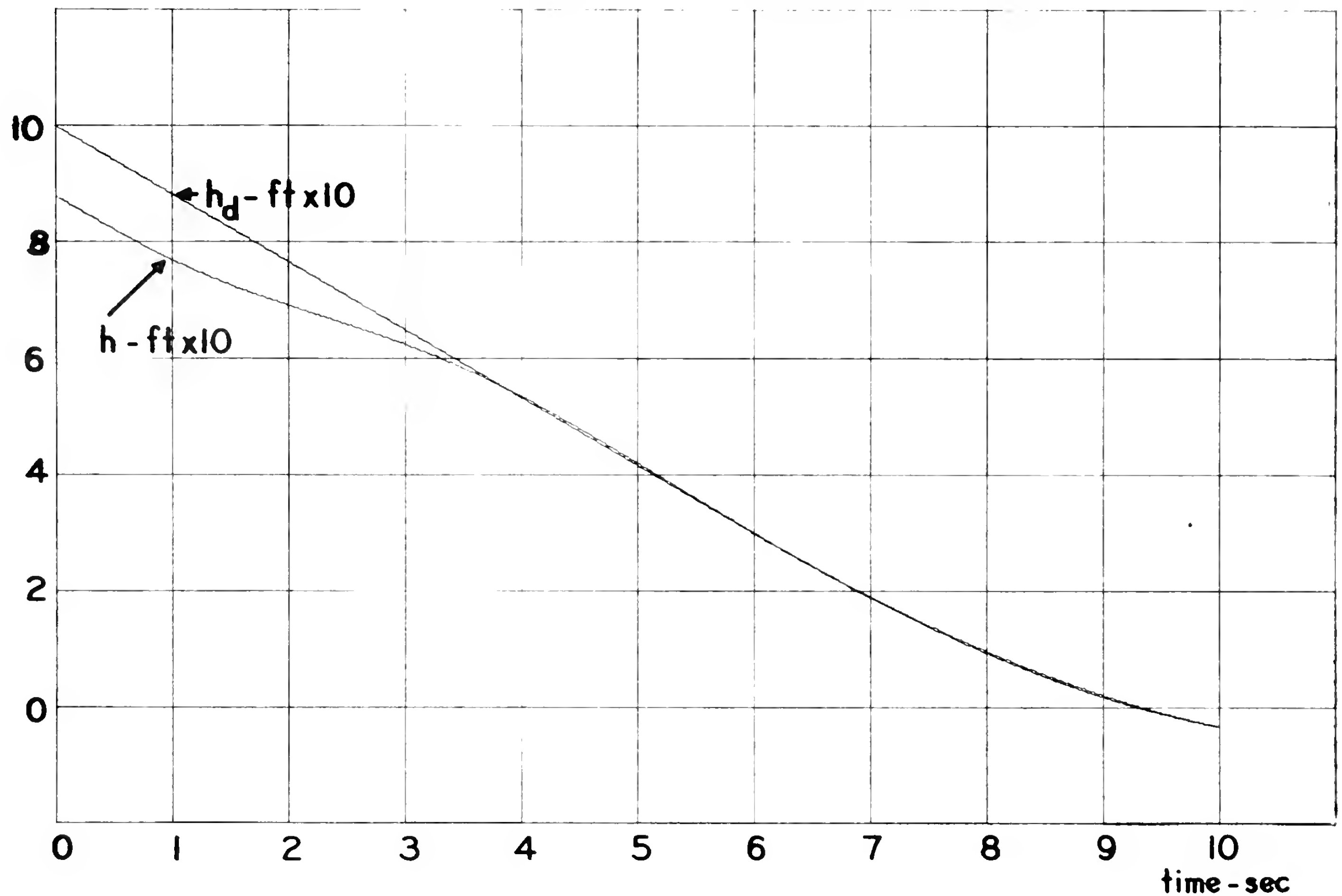

FIGURE 30
THE OPTIMAL AND DESIRED
ALTITUDE TRAJECTORIES FOR CASE IIC

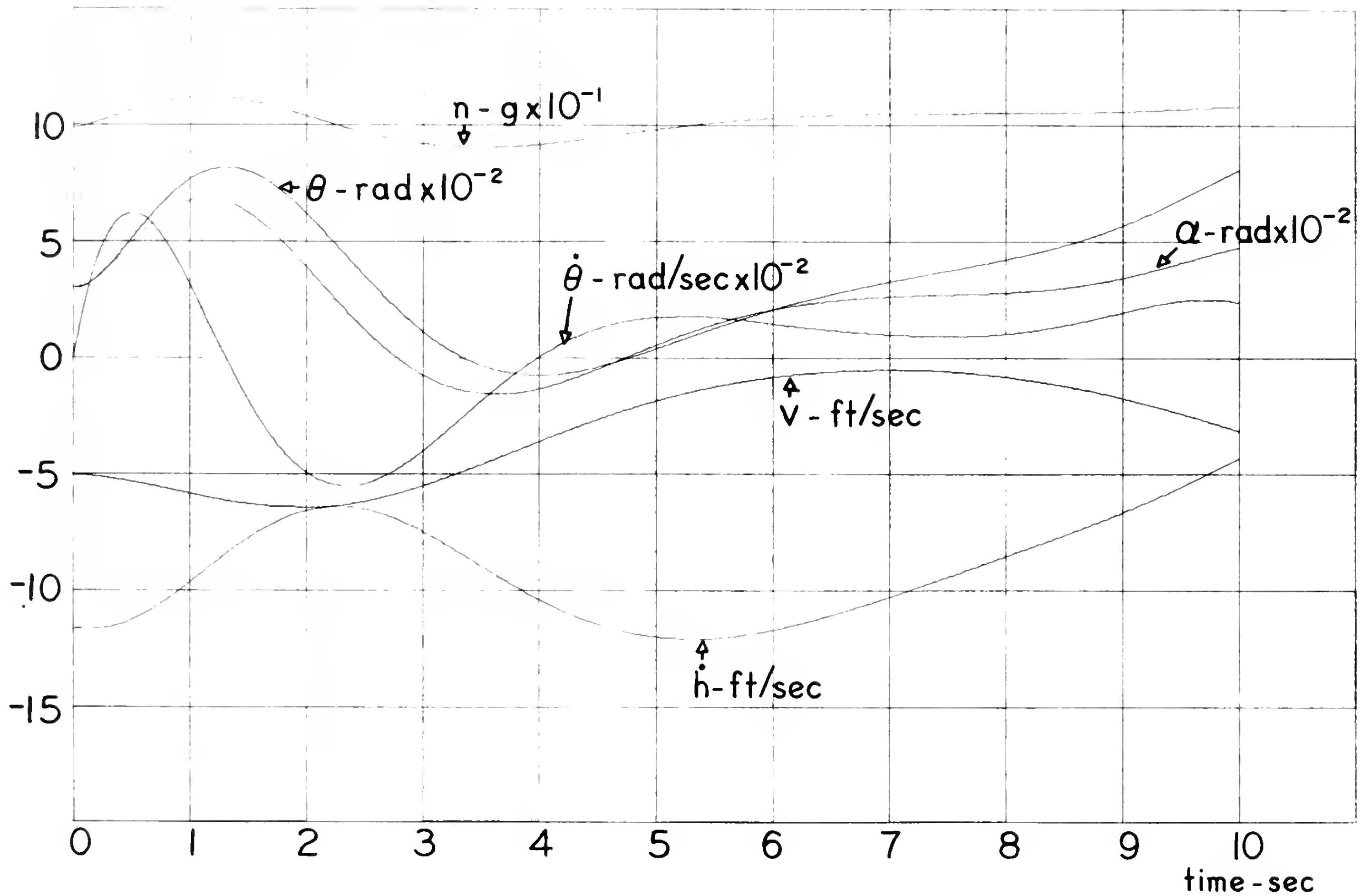

FIGURE 31
THE OPTIMAL v, α, θ, AND $\dot{\theta}$
HISTORIES AND n, $\dot{h}$ TRACES FOR CASE IIC

VIII. CONCLUSIONS

For the results presented in Chapter VII, the following conclusions are stated:

1. The feasibility of obtaining realistic results by formulating the all-weather landing problem as a linear tracking problem and applying optimal control techniques was demonstrated.

2. Satisfactory landings, using a simple mathematical model of the F-4J airplane, were accomplished in all cases investigated when the derived optimal control was applied.

3. Obtaining values for the elements of the weighting matrices, H, Q, and R, to produce admissible and realistic results was difficult because the state trajectory was sensitive to changes in these matrices. The process was problematical, since a large number of design specifications had to be satisfied.

Recommendations for logical extensions of this investigation follow:

1. Formulate the problem to include the control dynamics.

2. Formulate the problem to include measurement noise and wind effects.

3. Investigate application of sub-optimal control techniques to the problem.

4. Investigate the possibility of using the sensitivity of the state trajectory to changes in the weighting matrices as a basis for developing a method of obtaining representative values for the elements of these matrices.

APPENDIX A REPRESENTATIVE DATA FOR THE F-4J AIRPLANE IN THE LANDING CONFIGURATION

Gross Weight	32,500 lb
Equilibrium Velocity (v_o)	223 ft/sec
Stall Velocity (V_{stall})	195 ft/sec
Equilibrium Angle of Attack (α_o)	0.33 rad
Stall Angle of Attack (α_{stall})	0.49 rad
Limit Sink Rate at Touchdown	23 ft/sec
Approximate Thrust Available	
MIL engine operation	5,000 lb
A/B engine operation	12,000 lb
Aerodynamic Ground Effect	Negligible
Control Time Constants	
Elevator (τ_{δ_e})	0.10 sec
Thrust (τ_{δ_e})	0.15 sec

Dimensional Stability Derivatives

$X_v = -0.593 \times 10^{-1} sec^{-1}$

$X_q = 0.0$

$X_\alpha = 0.107 \times 10^{2} ft\ sec^{-2}$

$X_{\dot{\alpha}} = 0.0$

$X_{\delta_e} = 0.0$

$X_{\delta_T} = 0.950 \times 10^{-3} ft\ lb^{-1} sec^{-2}$

$Z_v = -0.259\ sec^{-1}$

$Z_q = 0.0$

$Z_\alpha = -0.827 \times 10^{2} ft\ sec^{-2}$

$Z_{\dot{\alpha}} = 0.0$

$Z_{\delta_e} = -0.132 \times 10^{2} ft\ sec^{-2}$

$Z_{\delta_T} = 0.289 \times 10^{-3} ft\ lb^{-1} sec^{-2}$

$M_v = 0.450 \times 10^{-3} ft^{-1} sec^{-1}$

$M_q = -0.430\ sec^{-1}$

$M_\alpha = -0.130 \times 10\ sec^{-2}$

$M_{\dot{\alpha}} = -0.260\ sec^{-1}$

$M_{\delta_e} = -0.279\ sec^{-2}$

$M_{\delta_T} = 0.0$

APPENDIX B SUMMARY OF NUMERICAL VALUES

STATE MODEL CONSTANT

$a_{11} = -0.59300\text{x}10^{-1}$

$a_{12} = 0.10700\text{x}10^{2}$

$a_{13} = -0.32172\text{x}10^{2}$

$a_{21} = -0.11628\text{x}10^{-2}$

$a_{22} = -0.37085$

$a_{41} = 0.75232\text{x}10^{-3}$

$a_{42} = -0.12036\text{x}10$

$a_{44} = -0.69000$

$a_{52} = -0.22300\text{x}10^{3}$

$a_{53} = 0.22300\text{x}10^{3}$

$b_{12} = 0.94667\text{x}10^{-3}$

$b_{21} = -0.59193\text{x}10^{-1}$

$b_{22} = 0.12979\text{x}10^{-5}$

$b_{41} = -0.27746\text{x}10$

$b_{42} = -0.33745\text{x}10^{-6}$

$c_{51} = -0.11676\text{x}10^{2}$

DIAGONAL WEIGHTING MATRIX CONSTANTS

CASE I

$h_{11} = 1.0$

$h_{22} = 1.0$

$h_{33} = 1.0$

$h_{44} = 2.0\text{x}10^{-3}$

$R = 5.0$

$q_{11} = 1.0\text{x}10^{-1}$

$q_{22} = 1.0\text{x}10^{-1}$

$q_{33} = 1.0\text{x}10^{-1}$

$q_{44} = 5.0\text{x}10^{-4}$

CASE II

$h_{11} = 5.0\text{x}10^{-5}$

$h_{22} = 5.0\text{x}10^{-1}$

$h_{33} = 5.0\text{x}10^{-1}$

$h_{44} = 1.0$

$h_{55} = 5.0\text{x}10^{-3}$

$r_{11} = 5.0$

$q_{11} = 1.0\text{x}10^{-5}$

$q_{22} = 1.0\text{x}10^{-1}$

$q_{33} = 1.0\text{x}10^{-1}$

$q_{44} = 5.0\text{x}10^{-1}$

$q_{55} = 5.0\text{x}10^{-4}$

$r_{22} = 5.0\text{x}10^{-10}$

```
C                         CCMPUTER PROGRAM
C
C
C
C                DERIVATICN OF THE OPTIMAL CCNTRCL
C            FOR AN ALL-WEATHER AIRPLANE LANDING SYSTEM
C
C                  NPGS MASTER THESIS, JUNE 1969
C
C
C                          MAIN PROGRAM
C
C
       REAL *8 ITITLE(12),KKDOT(20),KK(20),XXDOT(5),XX(5),
      *ITITL1(36),DTK,TOO,TFF,DTX
       REAL *4 A(25),B(12),H(5),Q(5),R(4),RRTF(5),AT(25),
      *K(2C),S(5),RRT(5),TEMP1(25),TEMP2(25),TEMP3(25),
      *QO(25),SDCT(5),KDCT(25),X(5),G(5),F(15),CC(5),P(15),
      *SVALS(5,1001),XVALS(5,1001),USTAR(2,1001),TBASE(1001),
      *BT(12),RI(4),LAB(30),HH(5),HHH(5),TEMP4(25),L1,L2,L3,
      *GBASE(600),ALFDCT(1001),ALTDOT(1001),ACCEL(1001),
      *KVALS(15,1001),XXX(1001),PSCAL(600)
C
C     GRAPHICAL OUTPUT DATA.
C       ITITL1(I) ARE GRAPH TITLES
C       LAB(I) ARE CURVE LABELS
C
       READ(5,524) (ITITL1(I),I=1,36)
       WRITE(6,5C5)
       WRITE(6,525)(ITITL1(I),I=1,36)
       READ(5,531) (LAB(I),I=1,15)
       READ(5,531) (LAB(I),I=16,30)
       WRITE(6,507)(LAB(I),I=1,15)
       WRITE(6,5C7)(LAB(I),I=16,30)
C
C     INITIALIZING DATA
C       NCASE IS CASE NUMBER(1=I,2=II)
C       NO IS NUMBER OF PLANT STATES
C       NC IS NUMBER OF CONTROLS
C       NSTK IS NUMBER OF INTERGRATION STEPS
C       DT IS INTEGRATION STEP SIZE
C       TO IS INITIAL TIME
C       TF IS FINAL TIME
C       L1,L2,L3 ARE CCNSTANTS CF DESIREC TRAJECTCRY EQUATION
C
       READ(5,5CC) NCASE,NO,NC,NSTK
       WRITE(6,501) NCASE,NO,NC,NSTK
       READ(5,5C2) DT,TO,TF,L1,L2,L3
       DTX=DT
       DTK=-DTX
       WRITE(6,5C3) DT,TO,TF,L1,L2,L3
       EPS=0.1E-04
       NT=0
       NK= NO*(NO+1)/2
       NKT=NK+NC
       NONO= NO*NC
       NCNC= NO*NC
       NSTK1=NSTK+1
       NSTK2 =NSTK+2
       NSTK3=NSTK/2
       NSTK4=NSTK3+1
       NSTK5=NSTK+3
       RAD=57.2958
       GRAV=32.1725
       GAMMA= -3.0/RAD
       VC= 223.0
       ALTO=100.C
C
C     TBASE IS TIME BASE FOR CCMPUTATION WHILE GBASE IS
C     TIME BASE FOR GRAPHICAL OUTPUT
C
       TBASE(1)=0.0
```

```
      DO 100 I=1,NSTK
  100 TBASE(I+1)=TBASE(I)+DT
      DO 101 J=1,NSTK4
  101 GBASE(J)= -TBASE(2*J-1)
C
C     PROBLEM DATA
C       A(I) ARE ELEMENTS OF A MATRIX
C       B(I) ARE ELEMENTS OF B MATRIX
C
      READ(5,504) (A(I),I=1,25)
      READ(5,504) (B(I),I=1,10)
      DO 102 I=1,NO
  102 CC(I)= 0.0
      CC(NO)=VO*GAMMA
      IF(NCASE.EQ.2) GO TO 105
      DO 103 I=1,NO
      B(I)= B(I+1)
      A(I)=A(I+6)
  103 A(I+4)= A(I+11)
      DO 104 I=1,NO
      A(I+8)=A(I+16)
  104 A(I+12)=A(I+21)
  105 WRITE(6,508)
      DO 106 J=1,NO
  106 WRITE(6,509)(A(I),I=J,NONO,NO)
      WRITE(6,510)
      DO 107 J=1,NO
  107 WRITE(6,509)(B(I),I=J,NONC,NO)
      WRITE(6,511)
      WRITE(6,516)  (CC(I),I=1,NO)
      READ(5,504) (X(I),I=1,NO)
      WRITE(6,523)
      WRITE(6,516) (X(I),I=1,NO)
      READ(5,504) (H(I),I=1,NO)
      READ(5,504) (Q(I),I=1,NO)
      READ(5,504) (R(I),I=1,NC)
      READ(5,504) (RRTF(I),I=1,NO)
      WRITE(6,512)
      WRITE(6,509) (H(I),I=1,NO)
      WRITE(6,509) (Q(I),I=1,NO)
      WRITE(6,509) (R(I),I=1,NC)
      WRITE(6,522)
      WRITE(6,516) (RRTF(I),I=1,NO)
C
C     FORM A TRANSPOSE, B TRANSPOSE, AND R INVERSE
C
      CALL MTRA (A,AT,NO,NO,0)
      CALL MTRA(B,BT,NO,NC,0)
      IF(NC.EQ.1) GO TO 109
      CALL MSTR(R,TEMP1,NC,2,1)
      CALL SINV (TEMP1,NC,EPS,IER)
      CALL MSTR(TEMP1,RI,NC,1,2)
      IF(IER-1) 110,108,108
  108 WRITE(6,514)
      GO TO 300
  109 RI(1)= 1.0/R(1)
  110 WRITE(6,526)
      DO 111 J=1,NO
  111 WRITE(6,509)(AT(I),I=J,NONO,NO)
      WRITE(6,527)
      DO 112 J=1,NC
  112 WRITE(6,509)(BT(I),I=J,NONC,NC)
      WRITE(6,528)
      WRITE(6,509)(RI(I),I=1,NC)
C
C     FORM FINAL TIME VALUES FOR K AND S
C
      DO 113 I=1,NO
  113 RRTF(I)=-RRTF(I)
      CALL MSTR(H,K,NO,2,1)
      CALL MPRD(H,RRTF,S,NO,NO,2,0,1)
      WRITE(6,515)
```

```
      WRITE(6,516) (K(I),I=1,NK)
      WRITE(6,517)
      WRITE(6,516) (S(I),I=1,NO)
      DO 114 I=1,NO
  114 RRT(I)=-RRTF(I)
C
C     FORM P MATRIX WHICH EQUALS B*RI*BT
C
      CALL MPRD(B,RI,TEMP1,NC,NC,0,2,NC)
      CALL MPRD(TEMP1,BT,TEMP2,NO,NC,0,0,NO)
      CALL MSTR(TEMP2,P,NO,0,1)
      WRITE(6,513)
      WRITE(6,516)(P(I),I=1,NK)
      CALL MSTR(Q,QQ,NO,2,0)
C
C     FORM KDOT AND SDOT AND INTEGRATE BACKWARDS
C
      DO 115 I=1,NK
  115 KK(I)=K(I)
      DO 116 I=1,NO
  116 KK(NK+I)=S(I)
      DO 131  L=1,NSTK
C
C     THE DESIRED TRAJECTORY IS INTRODUCED HERE.  IF TO,
C     TF, DT,NSTK OR DESIRED TRAJECORY ARE CHANGED, THE
C     FOLLOWING 9 CARDS MUST BE CHANGED ACCORDINGLY
C
      IF(L.GE.401) GO TO 117
      DELT= TBASE(401)-TBASE(L)
      TEMP= EXP(L1*DELT)-1.0
      RRT(NO)= (1.0/L1)*L2*TEMP - L2*DELT
      RRT(NO-1)=L3*DELT
      RRT(NO-2)=(L3/2.0)*DELT**2
      RRT(NO-3)=RRT(NO-2)-(L2/VO)*TEMP
      IF(NCASE.EQ.1) GO TO 119
      RRT(NO-4)= 0.0
      GO TO 119
  117 DO 118 I=1,NO
  118 RRT(I)=0.0
  119 DO 120 I=1,NK
  120 KVALS(I,L)=K(I)
      DO 121 I=1,NO
  121 SVALS (I,L)=S(I)
  122 CALL MPRD(K,A,TEMP1,NO,NO,1,0,NO)
      CALL MPRD (AT,K,TEMP2,NO,NO,0,1,NO)
      CALL MPRD (K,P,TEMP3,NO,NO,1,1,NO)
      CALL MPRD(TEMP3,K,TEMP4,NO,NO,0,1,NO)
      DO 123 I=1,NONO
  123 KDOT(I)= -TEMP1(I)-TEMP2(I)-QQ(I)+TEMP4(I)
      CALL MSTR(KDOT,TEMP1,NO,0,1)
      DO 124 I=1,NK
  124 KKDOT(I)= TEMP1(I)
      CALL MSUB (AT,TEMP3,TEMP4,NO,NO,0,0)
      CALL MPRD(TEMP4,S,TEMP1,NO,NO,0,0,1)
      CALL MPRD(Q,RRT,TEMP2,NO,NO,2,0,1)
      DO 125 I=1,NO
  125 SDOT(I)=-TEMP1(I)+TEMP2(I)
      DO 126 I=1,NO
  126 KKDOT(NK+I)= SDOT(I)
      SS=RKLDEQ(NKT,KK,KKDOT,TFF,DTK,NT)
      IF(SS-1.) 127,128,128
  127 WRITE(6,521)
      GO TO 300
  128 DO 129 I=1,NK
  129 K(I)= KK(I)
      DO 130 I=1,NO
  130 S(I)= KK(NK+I)
      IF(SS-1.) 127,122,131
  131 CONTINUE
      DO 132 I=1,NK
  132 KVALS(I,NSTK+1) = K(I)
      DO 133 I=1,NO
```

```
  133 SVALS(I,NSTK+1)=S(I)
C
C     WRITE K AND S VALUES
C
      NI=1
      IF(NK-10) 134,134,135
  134 NR=NK
       NTI=1
      NRI=0
      GO TO 136
  135 NTI=NK/10
      NR=10
      NRI=NK-NTI*10
  136 DO 138 M=1,NTI
      WRITE(6,518)
      DO 137 J=1,NSTK1
  137 WRITE(6,509) (KVALS(I,NSTK2-J),I=NI,NR)
      NI=NI+10
  138 NR=NR+10
      IF(NRI) 141,141,139
  139 WRITE(6,518)
      NRI=NRI+NTI*10
      DO 140 J=1,NSTK1
  140 WRITE(6,509) (KVALS(I,NSTK2-J),I=NI,NRI)
      IC=0
   10 READ (5,1000,END=999) CARD
 1000 FORMAT (20A4)
  141 WRITE(6,520)
      DO 142 J=  1,NSTK1
  142 WRITE(6,509) (SVALS(I,NSTK2-J),I=1,NO)
C
C     OBTAIN OPTIMAL CONTROL AND OPTIMAL TRAJECTORY
C
      NT=0
      DO 143 I=1,NO
  143 XX(I)=X(I)
      XXDOT(NO-3) = 0.0
      XXDOT(NO)= VO*GAMMA+VC*(X(NO-2)-X(NO-3))
      DO 144 I=1,NO
  144 HH(I)=0.0
      DO 157 L=1,NSTK
      DO 145 I=1,NO
  145 XVALS(I,L)=X(I)
C
C     ALTDOT IS THE RATE OF DESCENT AND ALFDOT IS THE
C     DERIVATIVE OF ALPHA
C
      ALTDOT(L)=XXDOT(NO)
      ALFDOT(L)=XXDOT(NO-3)
  147 DO 148 I=1,NK
  148 TEMP1(I)= KVALS(I,NSTK2-L)
      DO 149 I=1,NO
  149 TEMP2(I)= SVALS(I,NSTK2-L)
      CALL MPRD(FI,BT,TEMP3,NC,NC,2,0,NC)
      CALL MPRD (TEMP3,TEMP1,F ,NC,NO,0,1,NC)
      CALL MPRD (TEMP3,TEMP2,G,NC,NC,0,0,1)
C
C     HH(NO) IS EQUILIBRIUM ALTITUDE TRAJECTORY AND
C     HHH IS F*HH
C
      HH(NO)=ALTO -(VC*3.0/RAD)*TBASE(L)
      CALL MPRD(F,HH,HHH,NC,NO,0,0,1)
      CALL MPRD(F,X,TEMP3,NC,NO,0,0,1)
      DO 150 I=1,NC
  150 TEMP4(I)= -TEMP3(I)-G(I) +HHH(I)
      DO 151 I=1,NC
  151 USTAR(I,L)= TEMP4(I)
  152 CALL MPRD(A,X,TEMP1,NC,NO,0,0,1)
      CALL MPRD (B,TEMP4,TEMP2,NO,NC,0,0,1)
      DO 153 I=1,NO
  153 XXDOT(I)= TEMP1(I)+ TEMP2(I)+CC(I)
      SS=RKLDEQ(NO,XX,XXDOT,TOO,DTX,NT)
```

```
      IF(SS-1.) 154,155,155
  154 WRITE(6,521)
      GO TO 300
  155 DO 156 I=1,NO
  156 X(I)= XX(I)
      IF(SS-1.) 154,152,157
  157 CONTINUE
      DO 158 I=1,NO
  158 XVALS (I,NSTK+1) =X(I)
      ALTDOT(NSTK+1)=XXDOT(NO)
      ALFDOT(NSTK+1)=XXDOT(NO-3)
C
C     WRITE OPTIMAL CONTROL AND OPTIMAL TRAJECTORY
C
  160 WRITE(6,529)
      DO 161 J=1,NSTK
  161 WRITE(6,530)  TBASE(J),(USTAR(I,J),I=1,NC)
C     ACCEL IS THE NORMAL ACCELERATION
      DO 162 J=1,NSTK1
  162 ACCEL(J)=VO*(XVALS(NO-1,J)-ALFDOT(J))/GRAV +1.0
      WRITE(6,519)
      DO 163 J=1,NSTK1
  163 WRITE(6,530)  TBASE(J),(XVALS(I,J),I=1,NO),ALTDOT(J),
     *ACCEL(J)
C
C     GRAPHICAL OUTPUT-----SUBROUTINE DRAW,DEVELOPED
C     AT NPGS, IS USED TO CONSTRUCT THE FOLLOWING PLOTS
C     (SUBROUTINE AVAILABLE AT NPGS COMPUTER FACILITY)
C           1.   PLOT OF OPTIMAL CONTROL
C           2.   PLOT OF OPTIMAL AND DESIRED ALTITUDE
C                TRAJECTORIES
C           3.   PLOT OF VELOCITY, ALPHA, THETA,THETA DOT,
C                NORMAL ACCELERATION AND RATE OF DESCENT
C           4.   SOLUTION TO RICATTI EQUATION
C           5.   PLOT OF S VALUES
C
      DO 1 I=1,12
    1 ITITLE(I)=ITITL1(I)
      DO 7 I=1,NC
      IF(NC.EQ.1) GO TO 4
      GO TO(2,3),I
    2 MOD=1
      GRS=1.0
      GO TO 5
    3 MOD=3
      GRS=0.0001
       GO TO 5
    4 MOD=0
      GRS=1.0
    5 DO 6 L=1,NSTK3
    6 XXX(L)=USTAR(I,2*L-1)*GRS
    7 CALL DRAW(NSTK3,XXX,GBASE,MOD,0,LAB(I),ITITLE,
     *0.05,1.0,11,4,2,2,9,11,1,LAST)
      DO 8 I=7,12
    8 ITITLE(I)=ITITL1(I+6)
      DO 15 I=1,2
      GO TO (9,13),I
    9 MOD=1
C
C     IF DESIRED ALTITUDE TRAJECTORY IS CHANGED, NEXT
C     SIX CARDS MUST BE CHANGED ACCORDINGLY
C
      DO 10 L=1,600
   10 XXX(L)= ALTO+VO*GAMMA*TBASE(L)
      DO 11 L=601,NSTK1
      DELT=TBASE(L)-TBASE(601)
      TEMP=EXP(L1*DELT)-1.0
   11 XXX(L)=ALTO+VO*GAMMA*TBASE(L)+L2*TEMP/L1-L2*DELT
      DO 12 L=1,NSTK4
   12 XXX(L)= XXX(2*L-1)
      GO TO 15
   13 MOD=3
```

```
      DO 14 L=1,NSTK4
   14 XXX(L)= XVALS(NO,2*L-1)
   15 CALL DRAW(NSTK4,XXX,GBASE,MOD,0,LAB(I+2),ITITLE,
     *20.0,1.0,11,1,2,2,9,11,1,LAST)
      DO 16 I=7,12
   16 ITITLE(I)= ITITL1(I+12)
      DO 27 I=1,6
      GO TO (17,18,18,18,23,25),I
   17 MOD=1
      JJ=0
      IF(NCASE.EQ.1) GO TO 19
      GRS=0.01
      GO TO 21
   18 MOD=2
      GRS=1.0
      GO TO 21
   19 DO 20 L=1,NSTK4
   20 XXX(L)=0.0
      JJ=1
      GO TO 27
   21 DO 22 L=1,NSTK4
   22 XXX(L)= XVALS(I-JJ,2*L-1)*GRS
      GO TO 27
   23 DO 24 L=1,NSTK4
   24 XXX(L)= ACCEL(2*L-1)*0.1
      GO TO 27
   25 MOD=3
      DO 26 L=1,NSTK4
   26 XXX(L)= ALTDOT(2*L-1)*0.01
   27 CALL DRAW(NSTK4,XXX,GBASE,MOD,0,LAB(I+4),ITITLE,
     *0.05,1.0,11,4,2,2,9,11,1,LAST)
      DO 28 I=7,12
   28 ITITLE(I)=ITITL1(I+18)
      DO 29 I=1,NSTK4
   29 GBASE(I)= -TBASE(NSTK5-2*I)
      DO 37 I=1,NK
      DO 30 J=1,NSTK4
   30 PSCAL(J)= KVALS(I,2*J-1)
      CALL SCALE(NSTK4,PSCAL,60.0,GRS)
      GO TO (31,32,32,32,32,32,32,32,32,33,32,32,32,32,34),I
   31 MOD=1
      GO TO 35
   32 MOD=2
      GO TO 35
   33 IF(NCASE.EQ.1) GO TO 34
      GO TO 35
   34 MOD=3
   35 DO 36 L=1,NSTK4
   36 XXX(L)=KVALS(I,2*L-1)*GRS
   37 CALL DRAW(NSTK4,XXX,GBASE,MOD,0,LAB(I+10),ITITLE,
     *60.0,1.0,11,4,2,2,9,11,1,LAST)
      DO 38 I=7,12
   38 ITITLE(I)= ITITL1(I+24)
      DO 46 I=1,NO
      DO 39 J=1,NSTK4
   39 PSCAL(J)= SVALS(I,2*J-1)
      CALL SCALE(NSTK4,PSCAL,3.0,GRS)
      GO TO (40,41,41,42,43),I
   40 MOD=1
      GO TO 44
   41 MOD=2
      GO TO 44
   42 IF(NCASE.EQ.1) GO TO 43
      GO TO 44
   43 MOD=3
   44 DO 45 L=1,NSTK4
   45 XXX(L)= SVALS(I,2*L-1)*GRS
   46 CALL DRAW(NSTK4,XXX,GBASE,MOD,0,LAB(I+25),ITITLE,
     *3.0,1.0,11,4,2,2,9,11,1,LAST)
  300 CONTINUE
  500 FORMAT(4I10)
  501 FORMAT(    ///,T8,'NCASE=',I4,/,T8,'NO=',I4,/,T8,
```

```
     *'NC=',I4,/,T8,'NSTK=',I4)
  502 FORMAT(6E10.5)
  503 FORMAT(T8,'DT=',E13.5,/,T8,'TO=',E13.5,/,T8,'TF=',
     *E13.5,/,T8,'L1=',E13.5,/,T8,'L2=',E13.5,/,T8,
     *'TF=',E13.5)
  504 FORMAT(5E10.5)
  505 FORMAT('1',//)
  506 FORMAT(8E13.5)
  507 FORMAT(2X,15(A4,2X))
  508 FORMAT(///,T8,'A MATRIX',/)
  509 FORMAT(10E13.5)
  510 FORMAT(///,T8,'B MATRIX',/)
  511 FORMAT(///,T8,'C MATRIX',/)
  512 FORMAT(///,T8,'H,Q,AND R MATRICES(DIAG STORAGE)',/)
  513 FORMAT (///,T8,'P MATRIX (SYMETRIC STORAGE)',/)
  514 FORMAT(///,T8,'ERROR IN SINV',/)
  515 FORMAT(///,T8,'K(TF) (SYMETRIC STORAGE)',/)
  516 FORMAT(E13.5)
  517 FORMAT(///,T8,'S(TF) VECTOR',/)
  518 FORMAT (///,T8,'RICATTI COEF',/)
  519 FORMAT  (///,T8,'STATE SOLUTIONS',/)
  520 FORMAT (///,T8,' S COEF', /)
  521 FORMAT (///,T8,'ERROR IN RKLDEQ',/)
  522 FORMAT(///,T8,'RR(TF) VECTOR',/)
  523 FORMAT(///,T8, 'X(O) VECTOR',/)
  524 FORMAT(6A8)
  525 FORMAT(2X,6A8)
  526 FORMAT(///,T8,'A TRANSPOSE',/)
  527 FORMAT(///,T8,'B TRANSPOSE',/)
  528 FORMAT(///,T8,'R INVERSE(DIAG STORAGE)',/)
  529 FORMAT(///,T8,'OPTIMAL CONTROL',/)
  530 FORMAT(F6.3,8E14.5)
  531 FORMAT(15A4)
      END
C
C
C                           FUNCTION RKLDEQ
C
C
C     PURPOSE
C        THIS ROUTINE SOLVES A SYSTEM OF N FIRST-ORDER
C        ORDINARY DIFFERENTIAL EQUATIONS BY THE RUNGE-KUTTA-
C        GILL FOURTH-ORDER METHOD
C
C     DESCRIPTION OF PARAMETERS
C        N - NUMBER OF FIRST-ORDER EQUATIONS IN SYSTEM TO
C            BE SOLVED.(O.LE.N.LE.25)
C        Y - NAME OF LINEAR ARRAY OF LENGTH AT LEAST N,
C            IN WHICH SOLUTION VALUES WILL BE STORED.  THE
C            CALLING PROGRAM SHOULD SUPPLY INITIAL VALUES
C            BEFORE FIRST ENTRY.
C        F - NAME OF LINEAR ARRAY OF LENGTH AT LEAST N,
C            IN WHICH THE DERIVATIVES, COMPUTED IN USER'S
C            PROGRAM ARE STORED.
C        X - THE INDEPENDENT VARIABLE, WHICH IS ADVANCED
C            WITHIN RKLDEQ
C        H - THE INCREMENT FOR X,  WHICH MAY BE CHANGED AT
C            THE END OF ANY INTERVAL. (WHEN S=2.0)
C        M - AN INTEGER WHICH COUNTS THE NUMBER OF TIMES
C            ENTRY TO RKLDEQ HAS BEEN MADE DURING CURRENT
C            INTERVAL.  IT MUST BE INITALLY SET TO ZERO BY
C            USER BEFORE FIRST CALL OF RKLDEQ.  SUBSEQUENTLY
C            IT SHOULD NOT BE CHANGED BY USER.
C        S - A SWITCH TO BE TESTED BY USER UPON RETURN FROM
C            RKLDEQ.
C
C               IF S = 1.0, THE CALLING PROGRAM SHOULD NOW
C               COMPUTE VALUES OF F, USING CURRENT VALUES OF
C               X AND Y, AND RETURN TO RKLDEQ.
C
C               IF S = 2.0, THE END OF PRESENT INTERVAL HAS
C               BEEN REACHED.  USER SHOULD STORE AND/OR OUT-
```

```
C               PUT CURRENT X  AND/CR Y AND TEST FOR END OF
C                 COMPUTATION.
C
C
      FUNCTION RKLDEQ (N,Y,F,X,H,NT)
      REAL*8 Y,F,X,H,Q,H1,H2,H3,H6
      DIMENSION Y(1), F(1), Q(25)
      NT = NT +1
      GO TO (1,2,3,4),NT
    1 H1 = H
      H2 = H1 * .5D0
      H3 = H1 * 2.D0
      H6 = H1/6.D0
      DO 11 J =1,N
   11 Q(J) = 0.D0
      A = .5D0
      X = X + H2
      GO TO 5
    2 A = .2928932188134525
      GO TO 5
    3 A = 1.7071067811865475
      X = X + H2
      GO TO 5
    4 DO 41 I = 1,N
   41 Y(I) = Y(I) + H6 * F(I) -Q(I)/3.D0
      NT = 0
      RKLDEQ =2.
      GO TO 6
    5 DO 51 L = 1,N
      Y(L) = Y(L) + A *(H * F(L) -Q(L))
   51 Q(L) = H3 * A *F(L) +(1.D0-3.D0*A) *Q(L)
      RKLDEQ =1.
    6 RETURN
      END
C
C
C                          SUBROUTINE SCALE
C
C
      SUBROUTINE SCALE(NPTS,PTS,SCLE,GRS)
      DIMENSION PTS(600)
      RANGE=3.0*SCLE+SCLE/4.0
      MAX=ABS(PTS(1))
      DO 11 I=2,NPTS
      TMAX=ABS(PTS(I))
      IF(MAX-TMAX)10,11,11
   10 MAX=ABS(PTS(I))
   11 CONTINUE
      IF(RANGE-MAX) 12,22,13
   12 KFLAG=1
      IF(MAX.LE.(RANGE*10.0)) GO TO 14
      IF(MAX.LE.(RANGE*100.0)) GO TO 15
      IF(MAX.LE.(RANGE*1000.0)) GO TO 16
      IF(MAX.LE.(RANGE*10000.0)) GO TO 17
      IF(MAX.LE.(RANGE*100000.0)) GO TO 18
      GO TO 18
   13 KFLAG=2
      IF(MAX.GE.(RANGE*0.1)) GO TO 22
      IF(MAX.GE.(RANGE*0.01)) GO TO 14
      IF(MAX.GE.(RANGE*0.001)) GO TO 15
      IF(MAX.GE.(RANGE*0.0001)) GO TO 16
      IF(MAX.GE.(RANGE*0.00001)) GO TO 17
      IF(MAX.GE.(RANGE*0.000001)) GO TO 18
      GO TO 18
   14 TGRS=10.0
      GO TO 19
   15 TGRS=100.0
      GO TO 19
   16 TGRS=1000.0
      GO TO 19
   17 TGRS=10000.0
      GO TO 19
```

```
18 TGRS=100000.0
19 GO TO (20,21),KFLAG
20 GRS= 1.0/TGRS
   GO TO 23
21 GRS= TGRS
   GO TO 23
22 GRS=1.0
23 CONTINUE
   RETURN
   END
```

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13. ABSTRACT

Optimal control theory is used to derive a controller for the final phases of an all-weather landing in the McDonald Douglas F-4J airplane. The landing is formulated as a linear tracking problem by developing a mathematical model for the airplane which is linearized about an equilibrium flight condition, and by defining a desired state trajectory. Examples are presented which illustrate the performance of the system.

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